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Vindicating a Bayesian Approach to Confirming Miracles

A Response to Jordan Howard Sobel's Reading of Hume

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The evidentialist approach to theism relies on evidence to confirm (or disconfirm) the central tenets of theism.[1] Although the methodology involved in measuring confirmation is commonly associated with scientific theories, the machinations of confirmation theory can be applied to propositions of almost any sort, including religious claims.[2] One of the most fruitful approaches to evidentialism employs Bayes's theorem to demonstrate the cumulative effects of evidence to confirm theism. Using Bayesian methods to measure the confirmation for independent eyewitness testimony that a miracle has occurred is especially useful (as will be shown below). One of the most intriguing responses to this approach has been stated by Jordan Howard Sobel in *Logic and Theism*.[3] Sobel proffers a novel way to defend Hume's maxim, "That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the

Abstract: This paper defends a Bayesian approach to confirming a miracle against Jordan Howard Sobel's recent novel interpretation of Hume's criticisms. In his book, *Logic and Theism*, Sobel offers an intriguing and original way to apply Hume's criticisms against the possibility of having sufficient evidence to confirm a miracle. The key idea behind Sobel's approach is to employ infinitesimal probabilities to neutralize the cumulative effects of positive evidence for any miracle. This paper aims to undermine Sobel's use of infinitesimal probabilities to block a Bayesian approach to confirming a miracle.

1. The evidentialist approach is exemplified classically by William Paley, *Natural Theology* (1802; Oxford: Oxford University Press, 2006), and in contemporary works such as Richard Swinburne, *The Existence of God*, 2nd ed. (Oxford: Oxford University Press, 2004), and George Schlesinger, *Religion and the Scientific Method* (Dordrecht, Holland: D. Riedel, 1977).

2. Richard Swinburne, *An Introduction to Confirmation Theory* (London: Methuen & Co., 1973), 5: "Probability on evidence, or epistemic probability, may be ascribed to propositions of all kinds—moral, aesthetic, and theological propositions, as well as historical and scientific ones."

3. Jordan Howard Sobel, *Logic and Theism* (Cambridge: Cambridge University Press, 2004).

fact, which it endeavours to establish."[4] Below, I will show how the Bayesian approach to miracles can be used as a powerful approach to confirming a miracle. Then, I will explain and engage Sobel's interpretation of Hume's case against miracles.

The Bayesian Approach to Miracles

The Bayesian approach draws on a probabilistic analysis of the justificatory effects of testimony to demonstrate the confirmation of independent eyewitness testimony.[5] For the purposes of showing the effects of independent eyewitness testimony more clearly, I will use Bayes's theorem in its odds form.[6] The odds form of Bayes's theorem can be seen below, where I have taken the liberty of substituting some of the relevant equalities that have been shown elsewhere.[7] If there are n independent, equally reliable testimonies (T) to a single miracle (M), the odds form of Bayes's theorem represents the impact of the evidence thus:[8]

$$\frac{\mathrm{P}(M|T^n)}{\mathrm{P}(\sim M|T^n)} = \frac{\mathrm{P}(M)}{\mathrm{P}(\sim M)} \times \frac{\mathrm{P}(T|M)^n}{\mathrm{P}(T|\sim M)^n} \div \frac{\mathrm{P}(T)}{\mathrm{P}(T)} = \left(\frac{\mathrm{P}(M)}{\mathrm{P}(\sim M)}\right) \times \left[\frac{\mathrm{P}(T|M)}{\mathrm{P}(T|\sim M)}\right]^n$$

The ratio of the posterior probabilities of M and $\sim M$ is the product of the prior probability ratio and the likelihood ratio, which are shown in rounded and square brackets, respectively, in the expression on the right. Presumably the prior probability ratio will be strongly weighted against the occurrence of a miracle because $\mathrm{P}(M) \ll \mathrm{P}(\sim M)$. On the other hand, the likelihood probability ratio should favor the occurrence of testimony given that a miracle happened because $\mathrm{P}(T|M) > \mathrm{P}(T|\sim M)$. Although the prior probability ratio will

4. David Hume, *An Enquiry Concerning Human Understanding*, ed. Tom L. Beauchamp (1748; Oxford: Oxford University Press, 1999), 174.

5. Cf. George N. Schlesinger, "Miracles and Probabilities," *Noûs* 21 (1987): 219–32; *New Perspectives on Old-Time Religion* (Oxford: Oxford University Press, 1988), 100–19; "The Credibility of Extraordinary Events," *Analysis* 51 (1991): 120–6; Rodney D. Holder, "Hume on Miracles: Bayesian Interpretation, Multiple Testimony, and the Existence of God," *British Journal for the Philosophy of Science* 49 (1998): 49–65; Earman, *Hume's Abject Failure: The Argument against Miracles* (New York: Oxford University Press, 2000); Timothy J. McGrew, "Has Plantinga Refuted the Historical Argument?" *Philosophia Christi* 6 (2004): 21–2; Timothy and Lydia McGrew, "On the Historical Argument: A Rejoinder to Plantinga," *Philosophia Christi* 8 (2006): 34–7.

6. The odds form of Bayes's theorem is the way that some, such as Timothy McGrew and Schlesinger, prefer to represent the effects of eyewitness testimony on miracles (see citations in footnote 5).

7. See Holder, "Hume on Miracles," 54–6.

8. For ease of reading and presentation, I have removed the ubiquitous background knowledge from all probability notations.

most likely outweigh the likelihood probability ratio by a large magnitude (always rendering P(M) < P($\sim M$)), the testimonial evidence will increase exponentially with the number of independent testimonies. Testimony, especially when used in conjunction with other empirical evidence, can plausibly provide grounds for concluding that a miracle has occurred.

To illustrate the significance that independent testimony can have in confirming a miracle, consider the following scenario. Suppose that some miraculous event (M) in light of some (extratestimonial) empirical evidence (E) is likely to occur on the following ratio: P($M|E$) : P($\sim M|E$) = 100,000,000 : 1. Furthermore, suppose that there are ten independent eyewitnesses (W) where each one's testimony is more likely to occur at a ten to one ratio in favor of the miraculous event: P($W|M\&E$) : P($W|\sim M\&E$) = 10 : 1. When applied to the odds form of Bayes's theorem, the evidence provides the following results:

$$\frac{P(M|W^n\&E)}{P(\sim M|W^n\&E)}=\left(\frac{P(M|E)}{P(\sim M|E)}\right)\times\left[\frac{P(W|M\&E)}{P(W|\sim M\&E)}\right]^n=\left(\frac{1}{100{,}000{,}000}\right)\times\left[\frac{10}{1}\right]^{10}=\frac{100}{1}$$

The Bayesian framing of the evidence for miracles provides a clear and precise method for measuring the epistemic worth of this evidence. Of course, establishing the probability assignments for a Bayesian approach to confirm a miracle will vary from miracle to miracle and will depend largely on how one assesses the empirical data. For these reasons, the Bayesian approach as such is neutral with respect to the outcome of a debate on miracles since it allows the evidence to determine the outcome.

Sobel's Humean Objection

Sobel has raised a serious challenge to the Bayesian approach to confirming a miracle in his impressive book, *Logic and Theism*. Sobel suggests that the best way to take the Humean maxim against miracles is to read Hume as saying that the prior probability that a miracle will occur should be some infinitesimal number.[9]

9. An anonymous referee for this journal showed me that it is difficult to pin down whether Sobel is merely trying to offer a charitable description of Hume's argument against miracles or if Sobel himself accepts and endorses these criticisms. E.g., Sobel writes on p. 337 of *Logic and Theism*, "What follows is a *reconstruction of Hume's ideas* regarding full and certain proofs of various strengths. I do not *propose* that when a person 'perceives' events to be necessarily connected, and their separation is naturally impossible, his credences should be represented by a probability function that assigns a positive infinitesimal to their separation." But on the next page, Sobel then says, "This treatment accords to 'views' of natural impossibilities and necessities a kind of resilience that seems appropriate." For these reasons, this article will focus

> For Hume, M asserts what would in a person's view would be a miracle, only if M is logically possible and there is what Hume would term a 'proof' for this person against M that has moved him to view it as *naturally impossible*. For a quantitative gloss on Hume's idea of a miracle, I say that there *is* a 'proof' for a person against M *if*, for this person *P(M)< i*, for some positive *infinitesimal i*, and that such an equality holds for a person *if and only if* there is, for this person, such a 'proof' against M, *and no such 'proof' for M*. A 'firm and unalterable contrary experience' provides a person with such a proof against M if and only if *it has in fact given rise (causal, not justificational, notion) in this person to a credence M that is represented by such an* EXTRAORDINARILY *small number*.[10]

Central to understanding Sobel's proposal is his use of infinitesimal numbers. An infinitesimal number is an infinitely small number such that its absolute value will be smaller than any positive real number. A number x is infinitesimal if and only if for every integer n, $| nx | < 1$. Furthermore, $| 1/x |$ is larger than any real number. Infinitesimal numbers are not members of the set of real numbers; they are hyperreal numbers. Sobel explains that

> Positive infinitesimals are 'just the numbers' for contemplated transgressions of 'laws of nature' that, without being absolutely improbable, are 'less than probable' as these laws are themselves 'more than probable': Transgressions of laws can be *less than n-probable for every standard real n greater than 0, though not 0-probable*; the laws themselves can be *more than n-probable for every standard positive real less than 1, without being 1-probable*.[11]

Sobel maintains that Hume's position should affirm that when one's evidence for a miracle (M) is some positively probable evidence (E), and (E) is not taken to be a natural impossibility, then $P(M|E) \approx P(M)$. Since the prior probability that a miracle will occur is some infinitesimally small number, the degree of confirmation that E will confer on M is so minute that the $P(M|E)$ will be about the same as the $P(M)$. For this reason, the only kind of evidence that can confirm a miracle would be evidence that is as miraculous as the miracle in question. Thus, if a miracle's occurring is infinitesimally improbable (that is, infinitesimally close to zero), then the evidence that would be necessary to confirm the miracle would need to be infinitesimally probable (that is, infinitesimally close to one). In cases of testimony and multiple independent testimonies, Sobel explains that the infinitesimal improbability that a miracle will occur is "overcome by testimony, *only if* the falsehood

on Sobel's interpretation of Hume's criticism without assuming that Sobel accepts or does not accept Hume's criticism as well.

10. Sobel, *Logic and Theism*, 338 (emphasis in original).

11. Ibid.

of the testimony is 'infinitesimally improbable,' and similarly for *bodies of independent testimony*. . . ."[12]

In light of applying infinitesimal probabilities to miracles, Sobel has forged a clever way to defend Hume's maxim, "That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavours to establish."[13] Sobel's analysis would also vindicate Hume's standard of testimonial evidence for a miracle: "If the falsehood of his testimony would be more miraculous than the event which he relates, then, and not till then, can he pretend to command my belief or opinion."[14]

It is important to realize the significance of Sobel's interpretation of Hume's criticism. If this reading of Hume correctly describes the probabilities for miracles and any possible evidence for them, then the prospect of confirming a miracle through typical finite evidence—such as testimony—is a hopeless undertaking. After all, the finite accumulation of infinitesimally small numbers is not *ever* going to sum to a non-infinitesimal number. By arguing that the probability that a miracle has occurred can be valued as a positive nonzero infinitesimal, Sobel attempts to render testimony, even multiple independent testimonies, insufficient to confirm a miracle *in principle*.

Hume's framework for assessing the probabilities of miracles and their evidence as understood by Sobel would undermine the prospects of using Bayesian methods to confirm a miracle. But how successful is the Humean criticism presented by Sobel? I believe that the Humean criticism fails to capture the probabilities involved accurately. In particular, I will present a semantic problem and a justification problem for Sobel's rendition of the Humean criticism.

First, there is the problem of providing an intelligible semantic interpretation to Sobel's syntax "infinitesimally small/large probabilities." In other words, it is not clear *what it means* to ascribe an infinitesimal probability to some event. Sobel assigns the intrinsic probability that a miracle will occur an infinitesimally small value and claims that convincing evidence for a miraculous event would need to be infinitesimally large. What do these probability assignments mean? For comparison, consider the "atom lottery" where an atom will be drawn at random from the entire universe. While this yields an extraordinarily large improbability for some specific atom winning the lottery (approximately $1/10^{80}$), the result is still a *finite probability* that can be surmounted by probabilistically finite evidence. To appreciate Sobel's proposed probability assignments, notice that the intrinsic probability that a miracle will occur will be lower than the improbability of some particular atom's being drawn in the atom lottery. Indeed, the infinitesimal probability

12. Ibid., 339.
13. Hume, *Enquiry Concerning Human Understanding*, 174.
14. Ibid.

that Sobel attributes to Hume's criticism is not even on the same magnitude of the probability involved in the atom lottery.

If infinitesimal probabilities cannot be compared with probabilities in the atom lottery, it becomes worrisome that there is no way to cash out Sobel's syntactic ascription. Perhaps Sobel might suggest that infinitesimal probabilities would be similar to the probabilities involved in a lottery with an infinite number of possible members. But if this is what he means, then he is up against the problem of normalization.[15] Normalizability is a feature of probabilities such that summing the complete set of disjoint alternatives will equal 1. Normalizability is what makes it possible to "carve up" the probability "space" for any event among its possible constituents rationally. When attempting to sum an infinite number of disjoint alternatives, the sum—if there is a sum—will be infinite. The untoward result is that one cannot assign meaningful probabilities to nonnormalizable events, like a lottery with an infinite number of possible outcomes. (Is it more probable one will win in the infinite lottery with one ticket or ten million tickets? Without being able to normalize, the probabilities are the same.)

So, if Sobel's probability syntax cannot be understood as a very small yet finite probability (such as the atom lottery), and it cannot be rendered meaningful under the description of an infinitely membered set of outcomes (such as an infinite lottery), then it seems that there is no way to give a meaningful interpretation to an infinitesimal probability.[16] Perhaps there is some halfway house between the probabilities involved in the atom lottery and the infinite lottery, but the onus rests on Sobel to show a clearer mathematical and probability-theoretic explanation of how infinitesimal probabilities are to be understood.

Aside from the problem of giving a meaningful interpretation to Sobel's syntax, there is a second problem: it is not clear that one could ever justify ascribing infinitesimal probabilities to any event.[17] On what grounds could one justify an infinitesimal probability assignment? Sobel seems to think the following does the trick:

> According to Hume, a person views a logical possibility as a miracle only if he views it as a violation of a law of nature, and so views it as a *natural impossibility*. We have such views. Hume considers them to be philosophically suspect and incapable of fully face-saving analyses in terms of ideas derived from experience, but he thinks that they

15. The normalizability problem is discussed in Timothy McGrew, Lydia McGrew, and Eric Vestrup, "Probabilities and the Fine-Tuning Argument: A Sceptical View," *Mind* 110 (2001): 1030–2.

16. For further discussion of infinitesimal probabilities see Timothy McGrew and Lydia McGrew, "On the Rational Reconstruction of the Fine-Tuning Argument," *Philosophia Christi* 7 (2005): 441–2.

17. I am grateful for Timothy and Lydia McGrew for suggesting to me several ways to make this criticism.

> are natural and indeed irrepressible 'views' for *everyone*, including skeptics such as himself when they are not 'engaged in their scepticism' (in which they are usually not engaged). There is, he might say, a sense in which 'we cannot do without them': He might say that though we do not for any theoretical or practical purposes *need* them, we cannot, psychologically, *avoid* them in out ordinary thinking. The proposal I am making for reading "Of Miracles" is that such 'views' (scare-quotes in deference to Hume's philosophical suspicions) be accorded distinctive treatment in a probabilistic representation of a credence-state, with all and only 'views' of natural impossibilities having *infinitesimal* probabilities in the representation. Similarly, all and only things 'viewed' as *natural necessities* will have probabilities that are, though less than, 'infinitely close' to 1.[18]

At the beginning of this passage, Sobel relies heavily on the notion of natural impossibility.[19] Perhaps, he means to suggest that everyone is compelled to recognize the causal closure of the physical universe. But this is not going to be sufficient to justify assigning an infinitesimal value to the probability that a miracle occurs. First, if causal closure holds for the physical universe, then the probability that a miracle will occur is zero. Of course, to know that the universe is causally closed, one would have to know that God does not exist or that even God cannot causally interact with the physical world. So, this route of justifying infinitesimal probabilities would turn out to be question-begging at best.

Maybe Sobel means that one has a probabilistically strong case for the causal closure of the physical world. I will call this inductive skepticism against miracles. Yet, even if such a strong case could be established, the result would surely not be so strong as to assign the probability that a miracle will occur an infinitesimal value. At best, such a proof would designate the probability that a miracle will occur to be some finite, although very low, probability. Furthermore, since the theist maintains that the miracle occurs by supernatural means (rather than by natural means), the fact that miracles are *naturally impossible* does not seem relevant at all. (Just as saying that rolling a three on a six-sided die is *evenly impossible*—impossible given that an even number is rolled—would have no significant bearing on rolling a three, unless one already thought all dice-rolls were restricted to the even numbers.) Only if one assumes that miracles must occur by natural means or that all events are natural events can one assign an infinitesimal probability to an event on the grounds that it is a "natural impossibility." To do this would beg crucial claims against one defending miracles.

Considering how Sobel's reading of Hume handles the famous case of the "Indian Prince" can reinforce the aforementioned point.[20] The Indian

18. Sobel, *Logic and Theism*, 338.
19. Cf. Sobel, *Logic and Theism*, 310–11.
20. Hume, *Enquiry Concerning Human Understanding*, 172.

Prince refused to believe that water could change to frost since in all of his experiences water never changed to frost. Whenever reports from abroad told of water changing to frost, the Indian Prince refused to believe them.

The Indian Prince presumed that water cannot turn to frost due to his uniform experience of the natural behavior of water. For the Indian Prince to believe that water can turn into frost (an event that he should count as a natural impossibility), Sobel would have Hume require the Indian Prince to acquire evidence that is infinitesimally close to a probability of 1. Yet, this seems overly stringent. Most people think that after one or two trustworthy testimonies the Prince would have reasonable grounds for believing that water can undergo this kind of transformation. Sobel's attempt to use inductive skepticism to justify assigning an infinitesimally low probability to a miraculous event has no principled way of demarcating the Indian Prince's apparent natural impossibility for water to change into frost from the belief that a miracle is naturally impossible. John Earman emphasizes how the Indian Prince example is embarrassing for this kind of inductive skeptical argument against miracles.

> If Hume's suggestion is that the prince's inductive leap is fallacious because it moves from experiences in one temperature range to a conclusion about an unexperienced temperature range, then it must be explained why the suggestion doesn't undermine all inductive reasoning. For *all* induction involves a leap from an observed range to an unobserved range, whether the range involves space, time, or a parameter such as temperature. . . . That embarrassment will always resurface in more complicated examples as long as the rule of induction yields a probability-one conclusion for a universal generalization from finite data. On the other hand, if Hume's straight rule of induction is modified so as to escape this embarrassment by assigning a probability less than 1, then Hume no longer has a "proof" against miracles, nor a principled distinction between miracles and marvels, and the way is opened for testimonies to establish the credibility of resurrections and the like.[21]

Another way to justify assigning an infinitesimal probability to a miraculous event might be grounded in psychological compulsion. Sobel claims that a Humean would hold that a miracle is a violation of a law of nature and would therefore believe a miracle to be a natural impossibility. These views, he writes, are "natural and indeed irrepressible"; "we cannot, psychologically, *avoid* them in our ordinary thinking."[22] Sobel adds that "a person's *first confidence* that events have natural causes—this presumption in its first appearance in a person's experience—is *natural*: It is not a conviction that *comes from* experience, but a conviction we are designed to *bring to* expe-

21. Earman, *Hume's Abject Failure*, 36–7.
22. Sobel, *Logic and Theism*, 338.

rience."[23] Given Sobel's own view that probabilities are subjective,[24] it is not surprising that he suggests that the most charitable way to understand Hume's criticism would adopt this approach.

On the subjectivist's view, the infinitesimal probability ascriptions for miraculous events are not rationally justified by appealing to psychological states. Rather, the advocate of subjective probabilities claims that one's spontaneous ascription of probabilities provides a causal account of how one arrives at such probability judgments. The only rational constraint on ascribing subjective probabilities will be that they are consistent so as to avoid Dutch book arguments. Perhaps Sobel intends to help Hume by suggesting that when one attributes infinitesimal probabilities to miraculous events, this probability assignment reflects one's subjective judgment and requires no rational justification (as long as it is consistent). Sobel says he is referring to a "causal, not justificational notion" in describing the experiential origin of infinitesimal credences.[25]

There is much that can be said against attempting to ground infinitesimal probabilities by appealing psychological causes. First, it is well-known that psychological intuitions about probabilities are not always well-founded. Gamblers often think that it is "their turn to win next time" even though such hopes are merely psychological and not rational. It may be that *some* people feel psychologically compelled to assign miracles an infinitesimally small probability, and they could also be mistaken. People have hunches, gut-feelings, unfounded intuitions, and other strong feelings that erroneously lead them to have false beliefs concerning probability assignments. Secondly, if Sobel is going to appeal to a subjective interpretation of probability to mend Hume's case against miracles, then he must disavow any claims to the effect that the subjectivist's judgment provides a rational justification for these probability assignments. Probability assignments turn out to be subjective reports of one's psychological states or degrees of confidence. But personal statements of this sort do not offer justifying reasons to think these judgments are correct. Nor do such judgments stand as a serious challenge to defending miracles by evidence. Thus, using a subjectivist interpretation to salvage infinitesimal probability judgments does not make Hume's case any more reasonable.

Since Sobel's interpretation of Hume fails to have a meaningful semantic interpretation, and it cannot justify ascribing an infinitesimally low probability to a miraculous event, his interpretation of Hume's criticism cannot

23. Ibid., 311 (emphasis in original).

24. The significance of Sobel's credence to subjective probabilities was recently highlighted in a book symposium on *Logic and Theism* where Richard Swinburne pressed Sobel on this point. Sobel's response ("To My Critics with Appreciation," *Philosophia Christi* 6 (2006): 257–64) gives an abbreviated explanation and defense of his understanding of subjective probabilities.

25. Sobel, *Logic and Theism*, 338.

be employed as a serious threat to the Bayesian approach to confirming a miracle. Since miracles cannot be given an infinitesimally low probability, then it remains possible in principle to accumulate enough finite evidence to confirm a miracle.[26]

26. I am grateful for Lydia McGrew, Timothy McGrew, Jonah Schupbach, and an anonymous referee for this journal for giving insightful comments that prodded me to fix some of the problems and clarify misunderstandings that were present in earlier drafts of this paper.