A Modal Model for Proving the Existence of God
Author(s): Robert E. Maydole
Reviewed work(s):
Published by: University of Illinois Press on behalf of the North American Philosophical Publications
Stable URL: http://www.jstor.org/stable/20013856
Accessed: 31/01/2013 20:10

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at http://www.jstor.org/page/info/about/policies/terms.jsp

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.
V. A MODAL MODEL FOR PROVING THE EXISTENCE OF GOD

ROBERT E. MAYDOLE

IN the first section of this paper, I employ an ontological type argument to show that the possibility of God implies, in a standard system of quantified modal logic, the existence and uniqueness of God. In the second section, I argue that the very idea of a supreme being is meaningful. In the third section, I use a cosmological type argument to prove that it is possible for God to exist. In the fourth and final section, I briefly discuss the plausibility of adopting the modal logic used in the first section.

I. THE POSSIBILITY OF GOD IMPLIES THE EXISTENCE OF GOD

In keeping with the idea of God as a supreme being, St. Anselm thought of God as being no greater than which can be conceived. This suggests two closely related, yet distinct things about what it means to be supreme. It suggests first that it is not possible for there to be something which is greater than a supreme being; and thus that a supreme being could not be greater than itself. It suggests secondly that it is not possible for there to be something distinct from a supreme being than which that supreme being is not greater; and thus that co-supremity among distinct beings is impossible.

Let \( \sim \), \( \bullet \), \( \forall \), \( \exists \), \( \Box \), \( \Diamond \), \( \Omega \), \( (3y) \), and \( (\forall y) \) stand for "not," "and," "or," "if then," "if and only if," "it is necessary that," "it is possible that," "there exists a y," and "for every y," respectively. Let "\( Gxy \)" be short for "x is greater than y" and "\( x = y \)" for "x is identical with y." The idea of a supreme being \( x \) can then be formally expressed as follows:

\[
\sim \Omega(3y)Gy \bullet \sim \Omega(3y)(\sim x=y \bullet \sim Gxy).
\]

Let this be abbreviated by "\( Sx \)." That there is a supreme being is then expressed by "\((\exists x)Sx\);" and that it is possible for a supreme being to exist is expressed by "\( \Diamond(\exists x)Sx \)."

The proof that "\( \Diamond(\exists x)Sx \)" implies "\((\exists x)Sx\)" is carried out in an S5 system of quantified modal logic called "CM." CM is the modal extension of the natural deduction system of first order logic with identity of Copi's Symbolic Logic.\(^2\) \( \Box \) is the only modal primitive of CM, with \( \Omega \) defined as "\( \sim \Box \sim \)." The modal inference rules of CM are Necessity Elimination (NE: \( \Box A \rightarrow A \)) and Necessity Introduction (NI: \( A \rightarrow \Box A \)), provided that every assumption on which \( A \) depends is completely modalized.\(^3\)

The proof that "\( \Diamond(\exists x)Sx \)" implies "\((\exists x)Sx\)" can now be stated as follows:

1. \( \Diamond(\exists x)Sx \) premise
2. \( \sim (\exists x)Sx \)
3. \( \sim \Box \sim (\exists x)Sx \) 1, def. of \( \Omega \)
4. \( \Box \sim (\exists x)Sx \) 2, NI
5. \( \Box \sim (\exists x)Sx \bullet \sim \Box \sim (\exists x)Sx \) 3, 4 Conj.
6. \( (\exists x)Sx \) 3–5 IP.

That there is of necessity only one supreme being, God, turns out, interestingly, to be a theorem of CM, namely: \( \Box (\exists x)(x=y) \). Here is the proof:

\[
\begin{align*}
1. & \quad (Sx \bullet S\bar{z}) \\
2. & \quad (\sim \Diamond(3y)Gyx \bullet \sim \Diamond(3y)(\sim x=y \bullet \sim Gxy)) \\
& \quad (\sim \Diamond(3y)Gyx \bullet \sim \Diamond(3y)(\sim z=y \bullet \sim Gzy)) \\
& \quad 1, \text{ def. } \\Sigma
\end{align*}
\]

\(^1\) The proof could also be carried out in other S5 systems of modal logic, such as Massey's natural deduction system Q-M, and Hughes and Creswell's axiomatic system LPC + S5. Q-M is described in Appendix G of Gerald J. Massey's Understanding Symbolic Logic (New York, 1972), ch. 6.

\(^2\) Irving M. Copi, Symbolic Logic. 4th edition. [New York, 1973]. Because "\( = \)" does not figure essentially in the proof of "\((\exists x)Sx\)" from "\( \Diamond(3x)Sx \)" in the sense that any predicate could replace "\( = \)" without any alteration in the proof, a logic with identity is not actually required. Yet there are two reasons for using a first order logic with identity rather than a plain first order logic. The first is that "\( = \)" occurs in "\( Sx \)." The second and main reason is that "\( (\exists z)(z = x) \)" is a theorem schema of a first order logic with identity, but not of plain first order logic. The theoremhood of "\( (\exists z)(z = x) \)" figures in my reply to subsequently discussed objection to my proof of "\( \Diamond(\exists x)Sx \)" from "\( \Diamond(3x)Sx \)."

\(^3\) A wff is completely modalized if and only if every predicate letter and every occurrence of a variable of the wff occurs within the scope of a modal operator.
Another interesting theorem of CM is Anselm’s Principle, which says that perfection cannot exist contingently. Stated differently, this principle says that it is necessarily the case that a perfect or supreme being exists only if such a being necessarily exists: \(\Box(\exists x)Sx \Rightarrow \Box(\exists x)\Box Sx\). The proof is:

1. \(\exists x Sx\)
2. \(\Box(\exists x)Sx\)
3. \(\Box(\exists x)Sx \Rightarrow \Box(\exists x)\Box Sx\)
4. \(\Box(\exists x)\Box Sx \Rightarrow \Box(\exists x)Sx\)

The monk Gaunilo objected to St. Anselm’s ontological argument on the ground that if it proves the existence of a being no greater than which can be conceived, then it also proves the existence of any superlative, such as a perfect island. Might not Gaunilo raise a similar objection to my argument? We could not, he might ask, use the modal method of arguing to show, say, that the possibility of a perfect island, an island no greater than which can be conceived, implies the existence of a unique perfect island? The answer is “no.”

Let “Is” be short for “is an island.” Then the idea of an island x no greater than which can be conceived, one such that it is not possible for any island to be greater, can be expressed formally as follows:

\[Ix \sim \Diamond(\exists y)(Iy \cdot Gy)\]
\[I \sim \Diamond(\exists y)(Iy \cdot z=x \cdot z= Gy).\]

Let this be abbreviated by “Px.” “Px” says that x is a perfect island. Now “Is” does not occur within the scope of a modal operator in “~(3x)Px.” So “~(3x)Px” is not completely modalized, and we cannot use NI to infer “\(\Box(\exists x)\Box Sx\)” from “~(3x)Px” in order to prove that “\(\Box(\exists x)\Box Sx\)” implies “(3x)Px,” although we were able to make that kind of move in proving that “\(\Box(3x)Sx\)” implies “(3x)Sx”.

A disciple of Gaunilo might counterobject by claiming that if incomplete modalization prevents the inference of a perfect island from the possibility of a perfect island, then it also prevents the inference of a supreme being from the possibility of a supreme being. Parallel to the expression of “x is a perfect island” by “Px,” he might say, we should express “x is a supreme being” by “Sx” rather than by “Sz,” where “Sz” is short for

\[Bx \sim \Diamond(\exists y)(By \cdot Gy)\]
\[\sim \Diamond(\exists y)(By \sim z=y \sim Gy).\]

and “Bx” is short for “x is a being.” Since “\(\sim(3x)Sx\)” is not completely modalized, he would argue, we cannot use NI to infer “(3x)Sx” from “\(\Box(3x)Sx\)”.

But Gaunilo’s disciple overlooks the fact that “Bx” is logically superfluous in CM. To say that x is a being logically is the same as saying that there is something which is identical with x. In other words, “Bx” can be replaced by “(3z)(z=x).” But “(3z)(z=x)” is provable in CM, which makes “Sx” logically equivalent to “Sz.” “\(\Box(3x)Sx\)” implies “(3x)Sx” after all.

II. THE VERY IDEA OF A SUPREME BEING

The proof that “\(\Box(3x)Sx\)” implies “(3x)Sx” will be an empty formalism if the idea of a supreme being does not make sense. The main ingredient in this idea is the predicate “is greater than.” Is this predicate meaningful? Does it make sense, in other words, to compare things in terms of ontological greatness? Anselm writes as though it were self evident:

... if one observes the nature of things he perceives, whether he will or no, that not all are embraced in a single degree of dignity; but that certain among them are distinguished by an inequality of degree. For he who doubts that the horse is superior in its nature to wood, and a man more excellent than a horse, assuredly does not deserve the name of a man.

Hartshorne joins with Anselm thus:

And if you ask about the import of “greater,” the reply is, x is greater than y insofar as x is, and y is not, something “which it is better to be than not to be.” Greater thus means superior, more excellent, more worthy of admiration and respect.


5 St. Anselm, Monologium, ch. IV.

But it would not seem to help much to explicate "is greater than" in terms of "is superior to," "is more excellent than" or, as it sometimes is, in terms of "is more perfect than." For if we consider "is greater than" as puzzling, then these other terms are equally puzzling. This manner of explication may have been perfectly acceptable to Anselm and other medieval philosophers who viewed reality through neo-Platonic glasses as constituting an ordered whole, with things being graded according to the degree of perfection, excellence, and even existence they possessed. It is not perfectly acceptable to those of us who do not wear those same metaphysical glasses. What we require is an analysis of "is greater than" in terms which are clearly meaningful, familiar and, hopefully, free from particular metaphysical trappings.

Hartshorne's mention of "is greater than" in terms of "more worthy of admiration and respect" strikes me as a step in the right direction. Plantinga takes it a step further.

A man who displays such qualities as wisdom and courage is greater, so far forth, than one who does not. Furthermore, a cat let's say, is not as great a being as a man, in that the latter has properties of intelligence and knowledge that the former lacks. Of course one being might have wisdom and intelligence but little courage and another just the reverse; then it might be hard to say which, if either was the greater. But such qualities as wisdom, moral excellence, power, courage and the like are what we might call "great making" properties; the more of these a being has, the greater, all else being equal, it is.

Malcolm specifies an entire range of "great making" properties

If a housewife has a set of extremely fragile dishes, then as dishes they are inferior to those of another set like them in all respects except that they are not fragile. Those of the first set are dependent for their continued existence on gentle handling; those of the second set are not. There is a definite connection in common language between the notions of dependency and inferiority, and independence and superiority. To say that something which was dependent on nothing whatever was superior to ("greater than") anything that was dependent in any way upon anything is quite in keeping with the everyday use of the terms "superior" and "greater." Correlative with the notions of dependence and independence are the notions of limited and unlimited. An engine requires fuel, and this is a limitation. It is the same thing to say that an engine's operation is dependent on as that it is limited by its fuel supply. An engine that could accomplish the same work in the same time and was in other respects satisfactory, but did not require fuel, would be a superior engine.

Hartshorne, Plantinga and Malcolm cite obvious ways in which things may be compared in particular respects. Hartshorne tells us that we may compare things with respect to that which is more worthy of our admiration and respect. We frequently do claim, and meaningfully so, that one person is more worthy of our admiration and respect than another. The loving and charitable person is more worthy of our admiration and respect than the greedy millionaire who continues to swindle his fellows, not because he needs the extra goods, but simply because he wants to pad his already secure nest egg. Put differently, we could say that the loving and charitable person is greater than our greedy millionaire with respect to the property of being worthy of our admiration and respect; even though our millionaire might be greater with respect to the having of wealth.

Plantinga cites wisdom, intelligence, power, moral excellence, courage, and the like as great making properties. These are particular respects in which one thing may be said to be greater than another. Thus, x might be greater than y with respect to intelligence, but y might be greater than x with respect to courage.

Malcolm goes further still. He tells us in effect that one thing x may be greater than another thing y with respect to being less limited by or dependent on z. If y is limited by z and x is not, or if y is more limited by z than x is limited by z, then x will be greater than y with respect to being less limited by z. Conceivably, although Malcolm does not say so, one thing x might be greater than another thing y with respect to being less limited by z1, yet y may be greater than x with respect to being less limited by z2.

There are unquestionably many other ways by which to compare things in particular respects, each of which would generate a locution of the form "x is greater than y with respect to property P." Thus, we might say that Beethoven is greater than Montavanni with respect to composing. Or, Ty Cobb is greater than Lefty Grove with respect to hitting a baseball. The point of these examples is not so much that they are true, noncontroversial, readily decidable, or easy to secure agreement about. Rather, it is that they make sense, that they are meaningful. It may not be possible to get complete agreement about Beethoven being a greater composer than Montavanni. And even less agreement would probably be reached over

7 Alvin Plantinga, "God and Possible Worlds," a paper read at Davidson College on November 12, 1974.
whether Beethoven or Bach is the greater composer. In fact, we might want to say that Bach is not greater than Beethoven with respect to composing, and also that Beethoven is not greater than Bach with respect to composing. In doing so, we would not be contradicting ourselves, because "x is not greater than y with respect to composing" does not entail "y is greater than x with respect to composing." Yet, if it is meaningful to say that Beethoven is not greater than Bach with respect to composing—and it is—then it is also meaningful to say that Beethoven is greater than Bach with respect to composing. Although these comparison claims may be controversial, they are nonetheless meaningful. Their meaning is guaranteed by the fact that they are commonly used and understood by speakers of ordinary language.

I have dwelled so far on stressing that things may be compared in particular respects. What must be shown, however, is that things may be compared absolutely. That is, we want to show that it is meaningful to say that x is greater than y, without adding "in this or that respect."

To say that x is greater than y is not to compare x and y regardless or independently of how x and y may be compared in particular respects. On the contrary, x will be greater than y precisely because x is greater than y in various particular respects. Plantinga, as quoted above, is suggesting this when he says that more great making properties a being has, the greater, all else being equal, it is. Malcolm is suggesting the same when he says that one set of dishes is superior to another if the first is like the second in all respects save not being fragile.

The trouble with the suggestions of Plantinga and Malcolm is that things are rarely, perhaps never, the same in all relevant ways save the way they are being compared in a particular respect. Plantinga and Malcolm would have great difficulty, then, in even being able to say that one thing is absolutely greater than another. What is needed is an analysis of "is greater than" which takes into account not merely one, but all great making properties, or a sufficient subset thereof. We want to be able to say that x is greater than y because x possesses a certain combination of great making properties which y does not, even though y may possess certain great making properties which x does not. Moreover, we also want to say that a sufficient condition for x to be greater than y is that x possesses at least one combination of great making properties which y lacks, and that a necessary condition for x to be greater than y is that x possesses at least one combination of great making properties which y lacks. This suggests that the concept of being greater than should be expressed in disjunctive normal form.

Let "F(x, y, z)" be short for "x is greater than y with respect to z." Then the definition of "x is greater than y" should, as suggested, have the following form:

\[
F(x, y, P_y) \lor \ldots \lor F(x, y, P_m) \\
F(x, y, P_{y2}) \lor \ldots \lor F(x, y, P_{m2}) \lor \ldots \\
F(x, y, P_{y2}) \lor \ldots \lor F(x, y, P_{m2})
\]

where each "P_i" represents a great making property with respect which it is meaningful to compare x and y. We will also want each of these great making properties to be such as to guarantee that two things may be equally great, and that "is greater than" is irreflexive, anti-symmetric and transitive.⁹

I believe that power, courage, intelligence, knowledge, kindness, beauty, truthfulness, gentleness, lovingness, and spiritedness should be counted among the great making properties. But what the great making properties are exactly, and how the predicates that represent them should be conjoined in the disjunctions of the suggested definitional form of "is greater than" are questions beyond the scope of the present paper. Their answers would provide us with a specification of what "is greater than" actually means; and that is something that will have to wait until further evidence is acquired and analyzed concerning how "is greater than" functions in a wide variety of contexts, including the religious, the moral, and the aesthetic. Even though I am not presently in the position of being able to provide anything more than the definitional schema of "is greater than" we need not for that reason decline from saying that "is greater than" is in fact meaningful. Many phrases of our language are meaningful and understood, even when users of the language are not in a position to explicitly define those phrases. I think that the phrase "is greater than" is used and understood—hence, meaningful; and, thus, that the very idea of a supreme being makes sense.

III. THE POSSIBILITY OF GOD

We are interested in demonstrating that the statement "It is possible for God to exist" is true of the actual world. According to possible world semantics,

⁹ This formal analysis is not meant to preclude the possibility of providing an informal explication of what "is greater than" means. Indeed, it is quite conceivable that an informal explication may prove more satisfactory in the long run than a formal analysis.
one way of demonstrating that the possibility statement "\(\diamond A\)" for any statement "A" is true of the actual world is by constructing a deductively valid argument "\(B_1, \ldots, B_n/A\)" such that each of the premises "\(B_1, \ldots, B_n\)" is true in some possible world which is accessible to the actual world, even though one or more of these premises happens to be false of the actual world. That is, if "\(B_1, \ldots, B_n/A\)" is deductively valid and each premise "\(B_1, \ldots, B_n\)" is true of the same possible world \(W\), then "\(A\)" will also be true of \(W\). But if "\(A\)" is true of \(W\) and \(W\) is accessible to the actual world, then "\(\diamond A\)" is true of the actual world.

So, if we had a deductively valid argument for the existence of God, all the premises of which are true of some possible world \(W\) which is accessible to the actual world, we could then correctly infer both that God exists in \(W\) and that it is possible for God to exist in the actual world. Or if we had an argument for the existence of God which is not deductively valid, but which could be extended to a deductively valid argument by the addition of one or more premises, and if the premises of this new, extended argument are all true in some possible world \(W\) which is accessible to the actual world, then here too we could correctly infer both that God exists in \(W\) and that it is possible for God to exist in the actual world.

I believe that St. Thomas Aquinas' Third Way can be appropriately modified to prove that it is possible for God to exist. Aquinas argued as follows:

The third way is taken from possibility and necessity and runs thus. We find in nature things that are possible to be and not to be, since they are found to be generated and to be corrupted, and consequently, it is possible for them to be and not to be. But it is impossible for these always to exist, for that which can not-be at some time is not. Therefore, if everything can not-be, then at one time there was nothing in existence. Now if this were true then even now there would be nothing in existence, because that which does not exist, begins to exist only through something already existing. Therefore if at one time nothing was in existence, it would have been impossible for anything to have begun to exist; and thus now nothing would be in existence—which is absurd. Therefore, not all beings are merely possible, but there must exist something the existence of which is necessary. But every necessary thing either has its necessity caused by another, or not. Now it is impossible to go on to infinity in necessary things which have their necessity caused by another. Therefore, we cannot but admit

the existence of some being having of itself its own necessity, and not receiving it from another, but rather causing in others their necessity. This all men speak of as God.11

The first thing to get straight about this argument is what Aquinas means by necessity. He does not mean logical necessity. Rather, he suggests that a necessary being is one that cannot be generated and cannot be corrupted. In the idiom of possible worlds, a necessary being for Aquinas is one that exists in a possible world only if it neither begins to exist nor ceases to exist in that or any other possible world; it is a being which either eternally exists or eternally fails to exist in every possible world where it exists. But a necessary being in this sense need not exist in all possible worlds. I prefer to say that such beings are temporally-necessary. Beings which are not temporally-necessary will be called temporally-contingent. Thus, a temporally-contingent being is one which can either be generated or can be corrupted. Again, in possible world's language, a temporally-contingent being is one which either begins to exist in some possible world or ceases to exist in some possible world. It could happen, however, that in a given possible world a temporally-contingent being exists eternally.

We can express the ideas of temporal-necessity and temporal-contingency more precisely as follows. Let the expression "\(t_1 < t_2\)" be short for the expression "time \(t_2\) is later than time \(t_1\)," and let "\(\chi\)" be short for "\(x\) is realized (exists) at time \(t\)." The notion of being generated can then be defined thus:

\[
\text{Gen}(x) = \forall t_1 (\exists t_2 (t_1 < t_2 \land \chi t_2 \land \neg \chi t_1))
\]

The notion of being corrupted can be defined similarly:

\[
\text{Cor}(x) = \forall t_1 (\exists t_2 (t_1 < t_2 \land \chi t_2 \land \neg \chi t_1))
\]

And given these two notions, we can then define the concepts of being temporally-necessary and temporally-contingent, respectively.

\[
\text{TemNec}(x) = \forall t_1 ~ \text{Gen}(x) \land \Box \text{Cor}(x)
\]

\[
\text{TemCon}(x) = \forall t_1 ~ \Box \text{Gen}(x) \lor \Box \text{Cor}(x)
\]

As expected, "\((\forall x) (\text{TemNec}(x) \equiv \text{TemCon}(x))\)" is a logical truth. Moreover, for any possible world where "\((\forall v_1) (\forall v_2) (v_1 < v_2 \land v_1 < v_2 \land v_2 = v_1)\)" and "\((\forall x) ((\exists t) \chi t_2 \lor (\exists t) \neg \chi t_2)\)" are true, such as the actual world, "\((\forall x) (\text{TemNec}(x) \equiv \Box (\forall t) \chi t \lor (\forall t) \neg \chi t)\)" is also true.

The Third Way can now be modified and extended

---

10 A possible world \(W_1\) is accessible to possible world \(W_2\) if and only if every statement true of \(W_1\) is possibly true of \(W_2\). More intuitively, \(W_2\) is accessible to \(W_1\) just in case someone living in \(W_1\) can conceive of what \(W_2\) is like. For more on the accessibility relation, see Hughes and Cresswell, An Introduction to Modal Logic, especially pp. 75-80.

11 Thomas Aquinas, Summa Theologica, p. 3, art. 3.
to apply to a possible world which is accessible to but possibly distinct from the actual world. The argument runs thus:

(1) Some temporally-contingent being presently exists.
(2) There have been only finitely many temporally-contingent beings to date.
(3) Every temporally-contingent being begins to exist at some time and ceases to exist at some time.
(4) Everything which begins to exist at some time and ceases to exist at some time exists for a finite period of time.
(5) If everything exists for only a finite period of time, and there have been only finitely many things, then there was a time when nothing existed.
(6) If there was a time when nothing existed, then nothing presently exists (since things begin to exist only through something else which already exists, and if there was a time when nothing existed then nothing could begin to exist at any time thereafter).
(7) Everything which exists exists for some time or other.
(8) Everything has a sufficient reason for its existence either in itself or beyond itself.
(9) There cannot be an infinite regress of sufficient reasons (for failure to reach an explanation is not an explanation).
(10) Notemporally-contingent being is a sufficient reason for a temporally-necessary being.
(11) Every temporally-necessary being which is a sufficient reason for its own existence—one whose essence, as it were, is to exist—is a being without any limitations.
(12) A being without any limitations is necessarily greater than any other being.
(13) It is not possible for anything to be greater than itself.
(14) It is necessarily the case that for all x and y, if x is greater than y, then y is not greater than x. (The relation of being greater than is necessarily anti-symmetric.)

Therefore,

(15) There is a supreme being.

This argument is deductively valid. Its premises, moreover, are mutually consistent, and thus true of some possible world W. It is plausible to assume that

W is accessible to the actual world, since we are here and now conceiving of W from the actual world. So (15) is true of W.

Statements (1), (2), (3), (6) and (10) can be thought of as logically-contingent facts about W; whereas, (4), (5), (7), (8), (9), (11), (13) and (14) are self-evident analytic truths, and hence true in all possible worlds. Only (12) requires special justification.

Assume x is a being without limitations in W. Then x possesses every great making property in W. In particular, x possesses the property in W of not being limited in world W1 by anything. In other words, if x is a being without any limitations in W, then x possesses every great making property in W. But the property of not being limited in W1 is a great making property of W. So it is true in W that it is true in W1 that x is unlimited. But for any statement p, if it is true in world w that p is true in world b, then p is true in world b.12 Hence, x is unlimited in world W1. Now if x is unlimited in W1, then in W1, x is greater than any other being in W1; otherwise x would be limited by not possessing a great making property possessed by something else. Hence it is true in W1 that x is greater than every other being. Since W1 is an arbitrarily selected possible world, it follows that it is true in every possible world that x is greater than every other being. Consequently, it is necessarily the case that x is greater than every other being. So (12) is true in W.

IV. Logical Assumptions

The proof that God is possible only if actual was carried out in a very strong system of quantified modal logic. Such logics have been the subject of considerable controversy in recent years. It is reasonable, therefore, to request the certification of any particular system of quantified modal logic whenever that system is employed in a demonstration, especially if the logic is as strong as CM. The task of fully justifying the use of CM, however, far exceeds the scope of this paper. What I intend to do instead is direct my attention to a particularly important mode of reasoning upon which the rejection of CM might well be based.

CM has the Barcan Formula (BF) as a theorem.

(BF): □((∀x)□A) ⊃ □(∀x)A

So in order to be able to accept CM as a viable modal logic (BF) will have to be necessarily true, that is,

---

true in all possible worlds. Yet Plantinga and others have argued persuasively to contrary. According to Plantinga,

The proposition expressed by the Barcan Formula . . . is easily seen to be false at any rate not necessarily true. No doubt there is a possible world \( W^* \) where no material objects exist—a world \( W^* \) in which the only objects are such things as propositions, sets, numbers and God. Now such a thing as a set is essentially immaterial; for surely no set could have been a material object. It is therefore true that

\[ (38) \text{ Every set is essentially immaterial.} \]

This is not a merely accidental feature of our immaterial world; (38) is necessarily true, true in every world. Hence it is true in this world \( W^* \) of which we are speaking. And of course sets are not unique in this regard; the same goes for properties, propositions, and God. Hence

\[ (38') \text{ Everything is essentially immaterial.} \]

is also true in \( W^* \). But

\[ (39) \text{ Necessarily, everything is an immaterial object.} \]

is false in \( W^* \), in view of the possibility of worlds like \( \alpha \) [the actual world] in which there exist material objects.

Let us assume for simplification, but without doing an injustice to Plantinga's argument, that his immaterial world \( W^* \) contains only numbers. The argument then boils down as follows:

1. Everything is a number.
2. Numbers are necessarily (essentially) immaterial.
   \[ \therefore (3) \text{ Everything is necessarily immaterial.} \]
3. Everything is necessarily immaterial.
   \[ \therefore (4) \text{ Necessarily everything is immaterial.} \]

(1) is the constitutional assumption about \( W^* \). (2) is viewed as a logical truth, hence true in \( W^* \). (3) is said to follow from (1) and (2); (4) from (3) and (BF). Reductio conclusion: (4) is true of \( W^* \). But (4) is really false of \( W^* \); otherwise, the actual world would have to be exclusively immaterial, which it is not. Final conclusion: (BF) is neither true of \( W^* \), nor necessarily true.

Compelling though it may seem, this argument is not without its difficulties. In the first place, I have some reservation about the existence of a possible world whose domain is different from the domain of the actual world. Indeed, if (BF) is adopted, even tentatively, as a necessary truth of an \( S_5 \) structure of possible worlds, then precisely the same objects exist in every possible world, albeit with differing inessential attributes. So instead of rejecting (BF) outright, Plantinga ought to consider the rejection of his constitutional assumption about \( W^* \). However, let us assume for the purpose of argument that (1) is true of \( W^* \). More serious difficulties are present.

Is it really true that numbers are essentially immaterial? In other words, are numbers immaterial in every possible world where they exist? Number theory suggests not. It is true of course that any model of number theory drawn from the actual world will be immaterial. This is because there are only finitely many material objects in the actual world and that the domain of any model for number theory must be infinite. But number theory does not uniquely determine the objects of its models. First, the incompleteness results of Gödel and others tell us that if number theory has a standard model then it also has nonstandard models. Second, we know from set theory that there are even diverse ways of construing the "numbers" which make up the standard models of number theory. Third, and most importantly, number theory does not at all specify that the objects of its models must be immaterial. Indeed, a subset of a possible world with infinitely many material objects might well be a model for number theory, with certain material objects of that world being numbers. Imagine a possible world with infinitely many material objects represented by the dots below.

\[
\bullet \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \quad \cdot \cdot \cdot \quad \cdot \cdot \cdot \quad \cdot \cdot \cdot \quad \cdot \cdot \cdot \quad \cdot \cdot \cdot \quad \cdot \cdot \cdot \quad \cdot \cdot \cdot \\
1 \quad 2 \quad 3 \quad 4 \quad \ldots \quad n \quad \ldots
\]

The numbers of this world are the material, composite individuals consisting of the objects within the cluster. 1 is a particular material object. 2 is a material cluster of two objects other than 1. 3 is a cluster of three objects other than those comprising 1 and 2. And so on. Plantinga could argue, of course, that there might be material models of number theory, but that the objects of such a model are not really numbers. He would then have to specify how to individuate numbers independently of number theory. Until such a specification is forthcoming, Plantinga's case against (BF) remains tenuous.

13 Properly speaking, \( A \) is necessarily true in world \( W \) if and only if \( A \) is true in all possible worlds accessible to \( W \). Constant reference to accessibility will only serve to unnecessarily complicate the subsequent discussion, however.
I now turn to the question of whether Plantinga’s argument is valid. Does (3) follow from (1) and (2)? Let “Nx” be short for “x is a number” and “Mx” for “x is material.” The formal renditions of (1) and (3) are, respectively:

(1f) \( (\forall x)Nx \)
(3f) \( (\forall x)\Box \sim Mx \)

But how are we to translate (2)? There are three possibilities:

(2fa) \( (\forall x)\Box (Nx \supset \sim Mx) \)
(2fb) \( \Box (\forall x)(Nx \supset \sim Mx) \)
(2fc) \( (\forall x)(Nx \supset \Box \sim Mx) \)

Plantinga’s argument against (BF) is stated informally and does not involve, so far as I can ascertain, a choice by him of one of these three ways of saying that a number is necessarily or essentially immaterial. All that Plantinga tells us is that x has a property P essentially if and only if x has P in every possible world in which it exists.15 This, however, is insufficient for enabling us to choose among the above three ways of possibly construing (2). Yet the argument against (BF) crucially depends on which of these three alternatives we adopt.

(3f) is implied by neither (2fa) and (1f), nor by (2fb) and (1f). So if either (2fa) or (2fb) is the modal translation of (2) then Plantinga’s argument is invalid.

Davidson College

Received April 12, 1979

\footnote{15 \textit{Ibid.}, p. 60.}