10

The Ontological Argument

ROBERT E. MAYDOLE

Ontological arguments are deductive arguments for the existence of God from general metaphysical principles and other assumptions about the nature or essence of God. There have been three very significant developments in the history of ontological arguments. The first is the ontological argument developed by St Anselm of Canterbury in the eleventh century. The second is the argument sketched by Descartes in the late seventeenth century and completed by Leibniz in the early eighteenth century. And the third development consists of the numerous ontological arguments of the twentieth century that explicitly utilize modal logic, particularly those of Malcolm, Hartshorne, Plantinga, and Gödel. My chief aim in this chapter is to logically evaluate logical reconstructions of each of these six arguments. I shall also present and logically discuss two of my own explicitly modal ontological arguments.[1]

The logical evaluation of a logical reconstruction of an argument often requires that we explicitly identify assumptions that are only implicit in the author's original presentation of the argument. And in some cases, it might involve the inclusion of "plausible" philosophical principles that are consistent with the author's worldview, principles that strengthen the argument if we include them among the premises of the reconstruction. My *modus operandi* will be to make each of the arguments as strong as possible before critically evaluating them. Even though I shall try to remain reasonably faithful to the intent of the original author of each argument, my main objective will be logical instead of historical.

A good deductive argument should be valid and have true premises. And if it is to be convincing it should not beg the question. Ontological arguments are frequently the target of parodies, perhaps more so than any other argument in philosophy. So in addition to checking the arguments we discuss for validity, truth, and question begging, I shall also test their vulnerability to being refuted by some of the well-known parodies in the philosophical literature. Like many other philosophical arguments of note, ontological arguments stand or fall on the acceptability of some very high-level and well-entrenched principles of metaphysics and logic. And, as we shall see, some ontological arguments are logically much stronger than what first meets the eye.

1. It would not be possible for me to discuss all or even most of the ontological arguments in the history of philosophy in this chapter. One of the most comprehensive and fairest discussions of many of these is by Graham Oppy (1995).

The Blackwell Companion to Natural Theology Edited William Lane Craig and J. P. Moreland
 ISBN: 978-1-405-17657-6

1a. The Validity of Anselm's Ontological Argument

Anselm expresses his ontological argument of *Proslogium,* Chapter ll as follows:

> Hence, even the fool is convinced that something exists in the understanding, at least, than which nothing greater can be conceived. For, when he hears of this, he understands it. And whatever is understood, exists in the understanding. And assuredly that, than which nothing greater can be conceived, cannot exist in the understanding alone. For, suppose it exists in the understanding alone: then it can be conceived to exist in reality; which is greater.
>
> Therefore, if that, than which nothing greater can be conceived, exists in the understanding alone, the very being, than which nothing greater can be conceived, is one, than which a greater can be conceived. But obviously this is impossible. Hence, there is no doubt that there exists a being than which nothing greater can be conceived, and it exists both in the understanding and in reality. (1962, p. 8)

There are a few key ideas in this passage that require our attention before we present the main argument. First, Anselm understands the predicate "is greater than" to mean the same as "objectively better or more worthy than." It was commonplace for philosophers of the Middle Ages to order things into a great chain of being according to the degree to which they possess great-making properties, such as wisdom, power, goodness, and completeness, and existence-in-reality. What Anselm argues here is that God is an upper bound to the great chain of being. He makes this point very explicitly in his *Monologion*:

> Furthermore, if one considers the nature of things, one cannot help realizing that they are not all of equal value, but differ by degrees. For the nature of a horse is better than that of a tree, and that of a human more excellent than that of a horse . . . It is undeniable that some natures can be better than others. None the less reason argues that there is some nature that so overtops the others that it is inferior to none. (1998, p. 14)

Second, the conclusion of Anselm's argument is that something than which nothing greater can be conceived has the property of existence-in-reality. Now if something has the property of existence-in-reality, then it exists, period. But the property of existence-in-reality is only one kind of existence. The property of existence-in-the-understanding is another. Notions, concepts, ideas, thoughts, beliefs, and so on are the kinds of things that do or might have existence-in-the-understanding. And tables, persons, angels, numbers, forces, and so on, and God, are the kinds of things that do or might have existence-in-reality. We can identify the realm of things that have existence-in-the-understanding with the totality of mental things that actually exist in minds; and we can identify the realm of things that have existence-in-reality with the totality of nonmental things that actually exist in the world.[2] It is crucial, however, not to conflate existence-in-reality with existence

2. Deane translates the phrase "*esse in intellectu*" as "exists in the understanding" (Anselm 1962), and Charlesworth translates it as "exists in the mind" (Anselm 1998). But McGill translates it as "stands in relation to the understanding" (Anselm 1967) because, he contends, "exists in the understanding" and "exists in the mind" both have a Cartesian connotation of existing *in* a substantial place of some sort with*in* which certain mental phenomena occur, which is not, he says, what Anselm intended. "For him the intellect is the *intentional* phase of human being [*sic*]. It is man's active openness towards reality, toward real entities through "understanding" and towards possible entities through "conceiving". "It is never a self-enclosed place within which certain phenomena

generally. For Anselm, mental things that have existence-in-the-understanding exist just as much as nonmental things that have existence-in-reality, but in a different way.

Third, it is inconceivable that one and the same thing could have both existence-in-reality and existence-in-the-understanding. Things that have existence-in-reality are very different kinds of things from things that have existence-in-the-understanding. A table, for example, is different from the concept or idea of a table. Likewise, it impossible to think of God, even *qua* pure spirit, as having existence-in-the-understanding, even if God fails to have existence-in-reality or even if "the fool is convinced that something exists in the understanding, at least, than which nothing greater can be conceived." Either Anselm has been mistranslated or he misspoke and should have said that even the fool is convinced that the *concept* of something than which nothing greater can be conceived has existence-in-the-understanding; and instead of saying "whatever is understood, exists in the understanding," he should have said "the concept of whatever is understood has existence-in-the-understanding" and so on. That said, a neo-Platonist such as Anselm might hold that things that have existence-in-reality and things that have existence-in-the-understanding *might* share many or most of the same properties, including the property of it being inconceivable for something to be greater.

Fourth, it is clear that for Anselm the phrase "that than which nothing greater can be conceived" should be understood as a definite description which refers to the one and only one thing than which nothing greater can be conceived, God, even if there is no such being. Similarly, the proposition "the concept of whatever is understood has existence-in-the-understanding" should be understood in the context of his argument as saying that the concept of whatever a definite description that is understood refers to has existence-in-the-understanding. This is consistent with Anselm's belief that even though we cannot fully and adequately understand God, we do at least have a partial concept of God as a being than which nothing greater can be conceived, and thereby we can understand our referential talk *about* God. The presupposition is that some referring singular terms and definite descriptions could be free of existential import, and quantifiers should be allowed to range over possibilia (Girle 2003, chap. 4). Otherwise, some referential terms that refer to nonmental things, such as "God" and "the being than which nothing greater can be conceived," would have to refer to mental things that have existence-in-the-understanding, which makes no sense; or those referential terms would have to have to refer to things that have existence-in-reality, which would make the Anselmian ontological argument beg the question.

Finally, in order to be able to test our Anselmian argument for validity, it will be useful to present it in standard form and to express it in the language of quantification theory. We shall use the following lexicon:

$Ux =_{df}$ x is understood
$Sy =_{df}$ the concept of y exists-in-the-understanding
$Ex =_{df}$ x exists-in-reality
$Gxy =_{df}$ x is greater than y
$Fxy =_{df}$ x refers to y

occur, such as ideas and inferences . . ." (McGill 1967, p. 82). The first two translations are consistent with the assumptions and presuppositions of this chapter. McGill's is not. Anselm was both a Christian and a neo-Platonist, and he had to be committed as such to the existence of minds or souls *qua* substantial entities within which mental phenomena occur.

$Dx =_{df}$ x is a definite description
$d =_{df}$ the definite description "$(\iota x) \sim©(\exists y)Gyx$"
$g =_{df} (\iota x)\sim©(\exists y)Gyx$
$P(Y) =_{df}$ Y is a great-making property
$©\ldots =_{df}$ it is conceivable that . . .[3]

Here then is our logical reconstruction of Anselm's ontological argument:

A1 The definite description "that than which it is not conceivable for something to be greater" is understood. (*Premise*)

$$(Dd \,\&\, Ud)$$

A2 "That than which it is not conceivable for something to be greater" refers to that than which it is not conceivable for something to be greater. (*Premise*)

$$Fdg$$

A3 The *concept* of whatever a definite description that is understood refers to has existence-in-the-understanding. (*Premise*)

$$(x)(y)((Dx \,\&\, Fxy \,\&\, Ux) \supset Sy)$$

A4 It is conceivable that something is greater than anything that lacks a great-making property that it conceivably has. (*Premise*)

$$(x_1)(Y)[(P(Y) \,\&\, \sim Yx_1 \,\&\, ©Yx_1) \supset ©(\exists x_2)Gx_2x_1]$$

A5 Existence-in-reality is a great making property. (*Premise*)

$$P(E)$$

A6 Anything the concept of which has existence-in-the-understanding conceivably has existence-in-reality. (*Premise*)

$$(x)(Sx \supset ©Ex)$$

A7 It is not conceivable that *something* is greater than that than which it is not conceivable for something to be greater. (*Premise*)

$$\sim©(\exists y)Gyg$$

Therefore,

A8 That than which it is not conceivable for something to be greater exists-in-reality.

$$Eg$$

The following deduction proves that this argument is valid:
Deduction[4]

1.	Dd & Ud	pr
2.	Fdg	pr

3. The conceivability operator need not be made explicit for this argument, since the deduction shows that the argument is valid in nonmodal first-order quantification theory. However, I include it because it will be needed later, and it also improves readability.
4. See Appendix 1 for the rules of inference and so on of the logic used in this chapter.

3.	$(x)(y)((Dx \,\&\, Fxy \,\&\, Ux) \supset Sy)$	pr
4.	$(x_1)(Y)[(P(Y) \,\&\, {\sim}Yx_1 \,\&\, \copyright Yx_1) \supset \copyright(\exists x_2)Gx_2x_1]$	pr
5.	$P(E)$	pr
6.	$(x)(Sx \supset \copyright Ex)$	pr
7.	${\sim}\copyright(\exists y)Gyg$	pr
8.	$Fdg \,\&\, {\sim}\copyright(\exists y)Gyg$	2, 7 Conj
9.	$(\exists x)[{\sim}\copyright(\exists y)Gyx \,\&\, (z)({\sim}\copyright(\exists y)Gyx \supset z=x)$ $\&\, (Fdx \,\&\, {\sim}\copyright(\exists y)Gyx)]$	8, theory of descriptions
10.	${\sim}\copyright(\exists y)Gy\nu \,\&\, (z)({\sim}\copyright(\exists y)Gyz \supset z=\nu)$ $\&\, (Fd\nu \,\&\, {\sim}\copyright(\exists y)Gy\nu)$	9, EI
11.	${\sim}\copyright(\exists y)Gy\nu$	10, Simp
12.	$Fd\nu$	10, Simp
13.	$(P(E) \,\&\, {\sim}E\nu \,\&\, \copyright E\nu) \supset \copyright(\exists x_2)Gx_2\nu$	4 UI
14.	$(Dd \,\&\, Fd\nu \,\&\, Ud) \supset S\nu$	3 UI
15.	$(Dd \,\&\, Fd\nu \,\&\, Ud)$	1, 12, Simp, Conj
16.	$S\nu$	14, 15 MP
17.	$S\nu \supset \copyright E\nu$	6, UI
18.	$\copyright E\nu$	16, 17 MP
19.	${\sim}(P(E) \,\&\, {\sim}E\nu \,\&\, \copyright E\nu)$	13, 11 MT
20.	${\sim}((P(E) \,\&\, \copyright E\nu) \,\&\, {\sim}E\nu)$	19 Com, Assoc
21.	${\sim}(P(E) \,\&\, \copyright E\nu) \vee {\sim\sim}E\nu)$	20, DeM
22.	$P(E) \,\&\, \copyright E\nu$	5, 18 Conj
23.	$E\nu$	21, 22, DS, DN
24.	${\sim}\copyright(\exists y)Gy\nu \,\&\, (z)({\sim}\copyright(\exists y)Gyx) \supset z=\nu)$	10 Simp
25.	${\sim}\copyright(\exists y)Gy\nu \,\&\, (z)({\sim}\copyright(\exists y)Gyx) \supset z=\nu) \,\&\, E\nu$	23, 24 Conj
26.	$(\exists x)[{\sim}\copyright(\exists y)Gyx \,\&\, (z)({\sim}\copyright(\exists y)Gyx) \supset z=x) \,\&\, Ex]$	25 EG
27.	Eg	26, theory of descriptions

1b. The Truth of the Anselmian Premises

The first conjunct of A1 is true by definition. The second is introspectively true. For it does seem as though we understand the phrase "that than which it is not possible for something to be greater." As Anselm would say, "many people appear to understand it when they hear it – even the fool." This assumes, of course, that the relational predicate "is greater than" is meaningful and understood by those who claim that they understand it when they use it and hear it. Such an assumption could be challenged, although not lightly and not without good reason. Perhaps the absence of a plausible theory of great-making properties would constitute a challenge to the meaningfulness of "is greater than." However, the burden of proof of a claim that a word or phrase is meaningless must always fall on the challenger, especially when the word appears to be used with understanding by a great many people. And the predicate "is greater than" is just such a term. Indeed, many philosophers from Plato to the present, including most neo-Platonists, scholastics, and rationalists, believe that the things of the world can be ordered in terms of both ontological and/or normative greatness, the absence of a nearly complete and coherent theory of great-making properties notwithstanding.

A2 also is analytically true, given the presupposition that in order for the definite description "that than which it is not conceivable for something to be greater" to genuinely refer, that than which it is not conceivable for something to be greater must be a member of the domain of some possible world.

Anselm constructs an insightful, but somewhat muddled, argument for the bogus proposition "whatever is understood exists in the understanding" that we might be able to use in support of the more plausible A3.

> . . . When the fool hears mentioned a being than which a greater is inconceivable, he understands what he hears . . . Moreover . . . if this being is understood, it is in the understanding . . . For as what is conceived, is conceived by conception, and what is conceived by conception, as it is conceived, so is in conception; so what is understood, is understood by understanding, and what is understood by understanding, as it is understood, so is in the understanding. What can be clearer than this? (1962, p. 157)

This passage suggests either a deductive or an analogical argument for A3.
The deductive argument is:

1 Concepts are in that which conceives of concepts.
2 Whatever is in that which conceives of concepts has existence-in-the-understanding.
3 Therefore, the *concept* of whatever a definite description that is understood refers to has existence-in-the-understanding.

And the analogical argument is identical to the deductive argument, save for 2* in place of 2.

2* Having the property of existence-the-understanding is like having the property of being *in* that which conceives of concepts.

The deductive argument is clearly valid, and the analogical argument certainly appears to be inductively strong, with a degree of inductive strength that is proportional to the degree of likeness between conceiving and understanding. The common first premise is analytic. Premise 2 is true if the degree of likeness between conceiving and understanding is 100 percent; and 2* is true if that degree of likeness is high, which it surely is, even if less than 100 percent.

A4 is only implicit in Anselm's *Proslogium*. Yet it is so intuitively obvious that I know of nothing more intuitive and general from which we might infer it. We might, of course, classify it as analytic and say that it is built into the very meaning of being a great-making property that it potentially increases greatness when instantiated. But such a move would presuppose a better idea of greatness than I have been able to give. This is not to say that we do not know what greatness is, or that we do not know that some principles of greatness are true. Rather, it is that a good theory of greatness has yet to be constructed, as far as I know. And were such a theory to be developed, it would be tempting to view A4 as an axiom or first principle.

Let us now turn to A5, arguably the centerpiece of Anselm's ontological argument, but a proposition for which he never appears to argue. Note first that A5 does not say that existence *per se* is a great-making property. It says rather that existence-in-reality is great-making. There is a big difference. If existence is a property, it applies, for Anselm, to *both* the contents of minds (existence-in-the-understanding) and the contents of worlds. Existence-in-reality is narrower than existence, for there are things that exist (in-the-understanding) that do not exist (in-reality).

One interesting characteristic seemingly possessed by things that have existence-in-reality but not by things that only have existence-in-the-understanding is ontological completeness.[5] Something is *ontologically complete* if and only if every property or its negation is a member of the set of all its properties. This computer on which I am writing is ontologically complete because it possesses every possible property or its negation, including the property or its negation of containing a hydrogen atom that was formed a split second after the Big Bang. The set of properties possessed by this computer is also infinite. But my rather limited idea of this computer, howsoever accurate and robust, is surely finite. Nor does that idea include either the property or its negation of containing a hydrogen atom that was formed a split second after the Big Bang. And even if I now were to amend my idea of this computer to include such a property or its negation, there would always some other property and its negation, neither of which I attribute to my idea of the computer.

We can now formulate a plausible argument sketch for A5 that might appeal to an Anselmian:

1 Things that have existence-in-reality are ontologically complete. (*Premise*)
2 The property of being ontologically complete has the property of being great-making. (*Premise*)
3 For every property X and Y, X has Y if and only if everything that has X has a property that has Y. (*Premise*)
4 The property of being ontologically complete has the property of being great-making if and only if everything that has the property of being ontologically complete has a property that has the property being great-making. (*3, UI*)
5 Everything that has the property of being ontologically complete has a property that has the property being great-making. (*2,4, Equiv, Simp, MP*)
6 Hence, everything that has the property of existence-in-reality has a property that has the property of being great-making. (*1, 5, UI, HS, UG*)
7 The property of existence-in-reality has the property of being great-making if and only if everything that has the property of existence-in-reality has a property that has the property of being great-making. (*3, UI*)
8 Hence, the property of existence-in-reality has the property of being great-making. (*6, 7, Equiv, Simp, MP*)

A6 is the Anselmian cognate of the proposition that it is possible for God to exist, a premise common to many ontological arguments. One way of showing that A6 is true would be via ultrarealism. As a neo-Platonist, Anselm would have thought of the ontology of the world as partitioned in three ways: particulars, Forms, and minds. The Forms are instantiated either as the properties of the particulars or as universal ideas in minds. While Plato himself believed at times that the Forms had a greater degree of reality than the

5. This is not true of things that exist in the mind of God, all of which are ontologically complete.

particulars, which he relegates in *The Republic* to the shadows, the Platonist must still think of particulars together with the Forms as real existents. Anselm could therefore comfortably think of the world as divided into things that have existence-in-reality and things that have existence-in-minds. Forms and particulars exist-in-reality, and universal ideas exist-in-minds. We can therefore say that for an Anselmian, if the concept of something has existence-in-the-understanding, then a universal idea of it exists-in-a-mind. Or we can say that if the concept of something has existence-in-the-understanding, then a *it is possible that* a universal idea of it exists-in-a-mind.

Some medieval neo-Platonists, the so-called ultrarealists, believed that the order of thought (the universals) and the order of extramental particulars correspond exactly.[6] To each particular real chair, for example, there corresponds a universal idea of that chair, a so-called mental chair. The real chair and the mental chair instantiate exactly the same Forms, but in different mediums. Real chairs instantiate the Forms in matter. Mental chairs instantiate the Forms in-mind. Otherwise, real chairs and mental chairs have *exactly* the same nonexistential properties.[7]

It strikes me as a bit far-fetched to think that particulars and their corresponding mental replicas share exactly the same nonexistential properties. There is more to reality than what meets the mind's eye, and vice versa. But because conceivability generally outruns actuality, it is less far-fetched to think that for each conceivable mental replica of something, it is conceivable that there exists another real particular that has each and every nonexistential property of the replica. Call this "weak ultrarealism."

We can formulate two similar neo-Platonic and ultrarealistic arguments for A6. Let "Mx" $=_{df}$ "a mental replica of x exists-in-a-mind."

Argument 1

1 $(x)(Mx \supset \copyright Ex)$
2 $(x)(Sx \supset Mx)$
∴ $(x)(Sx \supset \copyright Ex)$

Argument 2

1 $(x)(Mx \supset \copyright Ex)$
2 $(x)(Sx \supset \copyright Mx)$
∴ $(x)(Sx \supset \copyright Ex)$

Argument 1 is valid in first-order quantification theory. Argument 2 is valid in an S4 modal-like extension of first-order quantification theory that licenses inferences from conceivable conceivability to conceivability. The same first premise of both arguments is quite weak and difficult to challenge. The second premise of Argument 1 could easily be challenged by a non-Platonist and a nondualist. Yet it would be harder to challenge the second premise of Argument 2, since it too makes a very weak claim.

A7 appears to be self-evident. Yet we can show that it is true in a couple of ways. First, the following argument is valid and both premises are logical truths:

6. Some historians of philosophy believe that Anselm was an ultrarealist (Copleston 1961, p. 35).
7. A nonexistential property is a property other than existence-in-reality and other than existence-in-the-mind.

1 $(Y)(z)[z=(\iota x)Yx \supset Yz]$
2 $g=(\iota x) \sim©(\exists y)Gyx$
$\therefore$ $\sim©(\exists y)Gyg$

Second, if we use Russell's Theory of Descriptions to eliminate the definite description "$(\iota x) \sim©(\exists y)Gyx$" from A7, we get:

A7a. $\sim©(\exists x)[\sim©(\exists y)Gyx \,\&\, (z)(\sim©(\exists y)Gyz \supset z{=}\, x) \,\&\, (\exists y)Gyx]$.[8]

If we then modestly assume that conceivability is equivalent to possibility, A7b becomes:

A7b. $\sim\Diamond(\exists x)[\sim\Diamond(\exists y)Gyx \,\&\, (z)(\sim\Diamond(\exists y)Gyz \supset z{=}\, x) \,\&\, (\exists y)Gyx]$.[9]

But A7b is logically true in even the weakest of modal logics. Therefore, A7 is true.

1c. On Whether Anselm's Ontological Argument Begs the Question

An argument begs the question just in case belief in the truth of the conclusion is included among the reasons for asserting the truth of the premises.[10] Some sound arguments sometimes beg the question. Consider the following valid argument:

Either 1 + 1 = 3 or God exists.
Not 1 + 1 = 3.
Hence, God exists.

While it is true that theists will believe that it is sound and nontheists believe that it is not, neither belief makes it so. Assume that no one believes that 1 + 1 = 3. If some theists believe that the argument is sound because they believe *qua* theists that the second disjunct of the first premise is true, then they beg the question. Likewise, if some nontheists believe that the argument is not sound because they believe *qua* nontheists that the second disjunct is false, then they too beg the question. Yet the argument would not beg the question if its proponent believed that the first premise is true for reasons that do not include the proposition that God exists. It would only be pointless.

William L. Rowe argues that Anselm's ontological argument begs the question by granting what it tries to prove (2001, pp. 39–41). According to Rowe, Anselm's ontological argument boils down to one that defines God as a greatest possible being, and also counts

8. The description "$(\iota x)\sim©(\exists y)Gyx$" occurs within the scope of "$\sim©$" and, clearly, has a secondary occurrence in A7. If we were to construe it as having a primary occurrence, then A7 would expand to "$(\exists x)[\sim©(\exists y)Gyx \,\&\, (z)(\sim©(\exists y)Gyz \supset z{=}x) \,\&\, \sim©(\exists y)Gyx]$," and Anselm's argument would thereby beg the question.
9. I realize that equating conceivability with possibility is controversial. But it is an issue that is beyond the scope of this chapter.
10. Strictly speaking, arguments do not beg the questions. Arguers do. Thus, an argument might beg the question for one person and not for another.

the property of existence-in-reality as great-making.[11] These two things, he correctly argues, imply that nothing that fails to exist-in-reality can be a greatest possible being; but they do not alone imply that a greatest possible being actually exists-in-reality. However, if we also assume that a greatest possible being possibly exists-in-reality, we can then infer that a greatest possible being actually exists-in-reality because no such possible being could fail to exist-in-reality and still be a greatest possible being, given that the property of existence-in-reality is great-making. This means, Rowe then concludes, that the assumption that a greatest possible being possibly exists-in-reality is "virtually equivalent" to the concluding proposition that it actually exists-in-reality. "In granting that Anselm's God is a possible thing, we are in fact granting that it actually exists . . . the argument begs the question: it assumes the point it is trying to prove" (Rowe 2001, p. 41).

Rowe is wrong on two counts. First, he equivocates on the word "grant." It can mean either "assume" or "implies." We assume (grant) the premises of an argument. And in granting these premises, we are in fact granting (implying) its conclusion if the argument is valid. Surely, the fact that the premises of an argument imply its conclusion does not mean that the argument begs the question. Second, the proposition that a greatest possible being possibly exists-in-reality is not at all equivalent to the proposition that it actually exists-in-reality, Rowe's use of the hedge word "virtually" notwithstanding. Indeed, the former does not even imply the latter, unless we assume that the other premises of Anselm's argument are logical truths. But one of the other premises is that the property of existence-in-reality is great-making, which is not a logical truth. Moreover, even if the proposition that a greatest possible being possibly exists-in-reality did imply the proposition that it actually exists-in-reality, the argument would beg the question only if the latter were given as a reason for believing the former.

The upshot is that Rowe's analysis does not show that his distilled version of Anselm's ontological argument begs the question. Nor is there any reason to think that the reasons I have given for believing the premises of the expanded version of the argument I have presented in sections 1a and 1b of this chapter include the proposition that the greatest conceivable being has the property of existence-in-reality. Therefore, we can confidently believe that it does not beg the question.

1d. On Parodies

A parody of an argument is a structurally similar argument with an absurd conclusion. There are two ways parodies can refute what they parody. First, if the parody has true premises and the same logical form as the argument parodied, then the argument parodied

11. Rowe's distilled version of Anselm's argument can be expressed schematically, thus:

1 Some possible object exemplifies the concept of God.
2 No object that fails to exist-in-reality could exemplify the concept of God. (Because God is defined as a being than which none greater is possible, and it assumed that the property of existence-in-reality is great-making.)
3 Every possible object either exists-in-reality or does not.
4 Therefore, God exemplifies the property of existence-in-reality.

must be invalid. Second, if both the parody and the argument parodied are valid, and the premises of the parody are at least as justifiable as the premises of the argument parodied, then the parody *refutes* the argument parodied because the argument parodied will also have to have at least one unjustifiable premise, and it will thereby fail to support its conclusion. In other words, if the premises of a valid parody are as justifiable as premises of the argument parodied, then it cannot be rational to believe that the premises of the argument parodied are true, for then it would be rational also to believe that the absurd conclusion of the parody is true. Conversely, if either the parody is invalid or some of its premises are arguably less justifiable than the respective corresponding premises of the argument parodied, then the parody *per se* fails to refute the argument parodied, and the argument parodied might well be sound.

Perhaps the most famous parody in the history of philosophy is the one formulated against Anselm's ontological argument by Gaunilo, a contemporary of Anselm, who reasoned that one could infer the absurd conclusion that a greatest conceivable island exists-in-reality from premises structurally similar to those of Anselm's argument. Let us add the following to our lexicon:

$Ix =_{df}$ x is an island
$i =_{df} (\iota x) \sim\copyright(\exists y)(Iy \,\&\, Gyx)$
$h =_{df}$ the definite description "$(\iota x)(Ix \,\&\, \sim\Diamond(\exists y)(Ix \,\&\, Gyx))$"

Then Gaunilo's parody is this:

G1	$Dh \,\&\, Uh$	pr
G2	Fhi	pr
G3	$(x)(y)((Dx \,\&\, Fxy \,\&\, Ux) \supset Sy)$	pr
G4	$(x_1)(Y)[(P(Y) \,\&\, \sim Yx_1 \,\&\, \copyright Yx_1) \supset \copyright(\exists x_2)Gx_2x_1]$	pr
G5	$P(E)$	pr
G6	$(x)(Sx \supset \copyright Ex)$	pr
G7	$\sim\copyright(\exists y)Gyi$	pr

Therefore,

G8 Ei

This parody is valid but not sound because G7 is false. So it fails to refute Anselm's argument. Replace G7 with G7a.

$$\text{G7a.} \quad \sim\copyright(\exists x_2)(lx_2 \,\&\, Gx_2 i)$$

Then the premises of the new parody are true if Anselm's premises are true; but the parody is not valid. So it fails to refute Anselm's argument. Replace G7 with G7a, and replace G4 with G4a.

$$\text{G4a.} \quad (x_1)(Y)[(P(Y) \,\&\, \sim Yx_1 \,\&\, \copyright Yx_1) \supset \copyright(\exists x_2)(lx_2 \,\&\, Gx_2x_1)]$$

This parody is valid but not sound because G4a is false.

Let “Lx_2x_1” be short for “x_2 is exactly like x_1 except for having Y in place of the negation of Y.” Replace G4 with G4b, and replace G7 with G7a.

$$\text{G4b.}\quad (x_1)(Y)[(P(Y)\ \&\ \sim Yx_1\ \&\ ©Yx_1) \supset ©(\exists x_2)(Lx_2x_1\ \&\ Gx_2x_1)].$$

Since “$©(\exists x_2)(Lx_2i\ \&\ Gx_2i)$” entails “$©(\exists x_2)(Ix_2\ \&\ Gx_2i)$,” the resulting parody is valid.

Or let “Kx_2x_1” be short for “x_2 is the same *kind* of thing as x_1.” Replace G4 with G4c, and replace G7 with G7a.

$$\text{G4c.}\quad (x_1)(Y)[(P(Y)\ \&\ \sim Yx_1\ \&\ ©Yx_1) \supset ©(\exists x_2)(Kx_2x_1\ \&\ Gx_2x_1)].$$

Since “$©(\exists x_2)(Kx_2i\ \&\ Gx_2i)$” entails “$©(\exists x_2)(Ix_2\ \&\ Gx_2i)$,” this resulting parody is also valid.

But do the last two parodies refute Anselm’s ontological argument? While it is certainly true that the addition of a great-making property to anything that conceivably has that property would result in something which would conceivably be greater than the first thing would be without that property, there is no guarantee I can think of for believing that the thing that would result from the addition of a great-making property would be exactly like the first thing sans that property, or even the same kind of thing as the first thing is without that property. In other words, the addition of a great-making property might change the nature of the thing it is added to. Take the property of existence-in-reality as an example. Things that exist-in-reality are very different from corresponding things that exist-in-the-understanding. A real table is different than the concept of a table, and a real tree is from the concept of a tree, and so on. So it is far from obvious that either G4b or G4c is true. But A4 is true intuitively. Consequently, neither of these parodies refutes Anselm’s argument.

Oppy suggests that Anselm’s ontological argument can be successfully refuted by parodies that purport to establish the existence of different kinds of devils:

> . . . Consider the formula ‘a being than which no worse can be conceived’. It seems that it would be worse if a very bad being existed both in the understanding and in reality than if it merely existed in the understanding. Consequently, it seems that if the Anselmian formula is understood as ‘a being than which no better can be conceived’ – then the Anselmian argument can be successfully parodied using this formula. (1995, p. 181)

Let us add the following notations to our growing lexicon:

$Vyx =_{df}$ y is more evil than x
$e =_{df} (\iota x) \sim ©(\exists y)Vyx$
$j =_{df}$ the definite description “$(\iota x)\sim\Diamond(\exists y)Vyx$”

Now replace “G” by “V,” “g” by “e,” and “d” by “j” in our reconstruction of Anselm’s ontological argument. Assuming that proposition “Ee” is absurd, Oppy’s putative *devils* parody is the result. Does it refute Anselm? Answer: only if premises O4 and O7 are arguably just as justifiable as A4 and A7, respectively.

$$\text{O4.}\quad (x_1)(Y)[(P(Y)\ \&\ \sim Yx_1\ \&\ ©Yx_1) \supset ©(\exists x_2)Vx_2x_1]$$
$$\text{O7.}\quad \sim ©(\exists y)Vye$$

Oppy suggests that O4 is true because an evil that exists-in-reality is more evil than it would be if it only existed-in-the-understanding. If the property of existence-in-reality is great-making, as it is assumed to be in the antecedent of O4, then an evil that exists-in-reality cannot be worse than any evil that does not. The addition of great-making properties make things better, not worse. So it is questionable that O4 is true, at least for the reason given by Oppy. Suppose, however, that an evil that exists-in-reality could be worse than one that does not. Then proposition "©(∃y)Vye" would presumably be true, and O7 false. The conclusion is that this parody also fails to refute Anselm's ontological argument.

2a. The Validity of the Ontological Argument of Descartes and Leibniz

Descartes expresses his ontological argument in "Meditation V," thus:

> It is certain that I no less find the idea of God, that is to say, the idea of a supremely perfect Being, in me, than that of any figure or number whatever it is; and I do not know any less clearly and distinctly that an [actual and] eternal existence pertains to this nature . . .
>
> I clearly see that existence can no more be separated from the essence of God than can the idea of its three angles equal to two right angles be separated from the idea of a [rectilinear] triangle.
>
> . . . from the fact that I cannot conceive of God without existence, it follows that existence is inseparable from Him, and hence that He really exists; not that my thought can bring this to pass, or impose any necessity on things, but on the contrary, because the necessity which lies in the thing itself, i.e., the necessity of the existence of God determines me to think this way . . . (1952, pp. 93–4)

He expresses it more clearly and succinctly in his "Arguments . . . in Geometrical Fashion":

> To say that something is contained in the nature of a concept of anything is the same as to say that it is true of that thing. But necessary existence is contained in the concept of God. Hence it is true to affirm of God that necessary existence exists in Him, or that God Himself exists. (1952, p. 132)

There are two key ideas in these passages. First, while Descartes says in "Meditation V" that *existence* is contained in the concept or essence of a supremely perfect being, it is clear from the context, and from the "Arguments . . . in Geometrical Fashion" that he really meant to say (or should have said) that God is a *necessary being*, where a necessary being is one that exists only if it exists necessarily.[12] Moreover, it would (should) have been obvious to him that the categorical property of *existence* (or nonexistence) cannot be included in the concept of anything without ultimately begging the question of its very existence (or non-existence).[13] Thus, for Descartes, it is really the conditional property of *existing necessarily*

12. The property of *existence* for Descartes is not split into *existence-in-reality* and *existence-in-the understanding*, as it is for Anselm.

13. We might call this the *Principle of Existential Noninclusion*. This principle does not preclude *asserting* that a supremely perfect being exists, only that existence must not be included in its concept or essence.

if existing at all, not the categorical property of *existence per se* (and not even the categorical property of having the property of *existence* necessarily), that is included in the concept or essence of a supremely perfect being. In other words, whether God exists or does not exist, He is not the kind of being who can exist contingently. Relatedly, a supremely perfect being possesses *supremity* necessarily.

Second, we need to clearly understand what is implied by the proposition that a certain property is contained in the concept or essence of something.[14] When we say, to use Descartes' own example, that the property of having its three angles equal to two right angles is contained in the concept or essence of a triangle, we imply that every triangle has the property of having its three angles equal to two right angles. In general, if property Y is contained in the concept or essence of something of kind X, then everything that is an X is a Y.

We are now in a position to logically reconstruct Descartes' ontological argument. Suppose

$Rx =_{df}$ x is supremely perfect
$Nx =_{df} (Ex \supset \Box Ex)$
$C(Y, X) =_{df}$ Y is included in the concept or essence of an X.

It would appear from the said quotations that Descartes' argument might be as follows:

D1 For every X and Y, if the property of being a Y is contained in the concept or essence of being an X, then necessarily everything that is an X is a Y.

$$(X)(Y)(C(\hat{y}[Yy], \hat{y}[Xy]) \supset \Box(z)(Xz \supset Yz)$$

D2 The property of *necessarily existing if existing at all* is contained in the concept or essence of a supremely perfect being.

$$C(\hat{y}[Ny], \hat{y}[Ry])$$

Therefore,

DC A supremely perfect being exists.

$$(\exists x)(Rx \,\&\, Ex)$$

It should at once be obvious, however, that this is an invalid argument. The only thing that relevantly follows from D1 and D2 is that everything that is supremely perfect necessarily exists if exists. But, if we add D3 and D4 as premises, then the argument is valid in S5 quantificational modal logic.[15]

D3 It is possible that a supremely perfect being exists.

$$\Diamond(\exists x)(Rx \,\&\, Ex)$$

14. Descartes vacillates between talking about the concept of things and talking about their essence. While concepts are generally thought of as subjective, and essences as objective, both will work in his ontological argument.

15. Although Descartes must have had sharp modal insights in order to be able to see that his enthymeme was valid, he could not have been aware of the exact modal principles used, since modal logic was not developed formally until the twentieth century.

D4 Necessarily, supremely perfect beings are necessarily supremely perfect.

$$\Box(x)(Rx \supset \Box Rx)$$

Then the following deduction proves that the argument D1, D2, D3, D4/∴DC is valid:

1	$(X)(Y)(C(\hat{y}[Yy], \hat{y}[Xy]) \supset \Box(z)(Xz \supset Yz)$	pr
2	$C(\hat{y}[Ny], \hat{y}[Ry])$	pr
3	$\Diamond(\exists x)(Rx \ \& \ Ex)$	pr
4	$\Box(x)(Rx \supset \Box Rx)$	pr
5	$C(\hat{y}[Ny], \hat{y}[Ry]) \supset \Box(x)(Rx \supset Nx)$	1 UI
6	$\Box(x)(Rx \supset Nx)$	2, 5 MP
7	$\Box(x)(Rx \supset (Ex \supset \Box Ex))$	6, df "N"
8	$(\Box(x)(Rx \supset (Ex \supset \Box Ex)) \ \& \ \Box(x)(Rx \supset \Box Rx)) \supset$ $\Box(x)((Rx \ \& \ Ex) \supset \Box(Rx \ \& \ Ex))$	theorem[16]
9	$\Box(x)(Rx \supset (Ex \supset \Box Ex)) \ \& \ \Box(x)(Rx \supset \Box Rx)$	4, 7 Conj
10	$\Box(x)((Rx \ \& \ Ex) \supset \Box(Rx \ \& \ Ex))$	8, 9 MP
11	$\Box(x)((Rx \ \& \ Ex) \supset \Box(Rx \ \& \ Ex)) \supset (\Diamond(\exists x)(Rx \ \& \ Ex) \supset \Diamond(\exists x)\Box(Rx \ \& \ Ex))$	theorem
12	$\Diamond(\exists x)(Rx \ \& \ Ex) \supset \Diamond(\exists x)\Box(Rx \ \& \ Ex)$	10, 11 MP
13	$\Diamond(\exists x)\Box(Rx \ \& \ Ex)$	3, 12 MP
14	$\Diamond(\exists x)\Box(Rx \ \& \ Ex) \supset \Diamond\Box(\exists x)(Rx \ \& \ Ex)$	theorem
15	$\Diamond\Box(\exists x)(Rx \ \& \ Ex)$	13, 14 MP
16	$\Diamond\Box(\exists x)(Rx \ \& \ Ex) \supset (\exists x)(Rx \ \& \ Ex)$	theorem
17	$(\exists x)(Rx \ \& \ Ex)$	15, 16 MP

2b. On the Truth of the Descartes–Leibniz Premises

D1 is analytically true. D2 and D4 are synthetic a priori metaphysical truths. D2 is true, according to Descartes, because the concept of *being supremely perfect* includes the having of all nonexistential perfections, and the property of *necessarily existing if at all* is a nonexistential perfection. But why is the conditional property of *existing necessarily if at all* a perfection (great-making)? Descartes says in "Meditation V" that existence ($\hat{y}[Ey]$) is a perfection (1952, p. 94).[17] If it is, then it should also be true that the categorical property of existing necessarily ($\hat{y}[\Box Ey]$) is a perfection. But property $\hat{y}[\Box Ey]$ entails the *conditional* property of *existing necessarily if at all*, $\hat{y}[Ey \supset \Box Ey]$. If we then accept that perfections entail only perfections, as I think we should, it follows that property $\hat{y}[Ey \supset \Box Ey]$ is a nonexistential perfection. Schematically,

1 $P(\hat{y}[Ey])$
2 $P(\hat{y}[Ey]) \supset P(\hat{y}[\Box Ey])$
3 $\Box(x)(\hat{y}[\Box Ey]x \supset \hat{y}[Ey \supset \Box Ey]x)$

16. The word "theorem" in these annotated proofs and deductions refers to theorems of the logic Q2S5. See Appendix 1.

17. The proposition that existence is a perfection is consistent with not including the property of existence in the concept or essence of a supremely perfect being, per the Principle of Existential NonInclusion.

4 $(Y)(Z)(P(Y) \,\&\, \Box(x)(Yx \supset Zx)) \supset P(Z))$
$\therefore$ $P(\hat{y}[Ey \supset \Box Ey])$

Or, as John Findlay has argued, supremely perfect beings are beings that by nature are worthy of worship, and beings that by nature are worthy of worship cannot exist contingently – because to be worthy of worship is to be absolutely perfect in every possible respect (1998, pp. 95–6). So supremely perfect beings must exist necessarily if at all.

Findlay also notes that a supremely perfect being cannot "possess its various excellences in some adventitious or contingent manner"(1998, p. 95). But the property of being supremely perfect is surely one of the excellences (great-makers). Therefore, if a being is supremely perfect, it must be supremely perfect necessarily. So D4 is true.

Descartes was clearly aware of the need to show that it is possible for a supremely perfect being (God) to exist (D3), even though he does not explicitly include it among the premises of the ontological argument he formulates in "Meditation V." Father Mersenne pointed it out to him in "The Second Set of Objections" that Descartes had attached to his first publication of the *Meditations*, and he responded to Mersenne in his appended "Reply," thus:

> But though we conceive of God only inadequately . . . this does not prevent its being certain that His nature is possible, or not contradictory; nor does it prevent our affirming truly that we have examined it with sufficient precision in order to know that necessary existence appertains to this same Divine nature. For all contradictoriness or impossibility is constituted by our thought . . . it cannot reside in anything external to the mind, because by the very fact that it is outside the mind it is clear that it is not contradictory, but is possible. Moreover, contradictoriness in our concepts arises merely from their obscurity and confusion. Hence it suffices us to understand clearly and distinctly those few things that we perceive about God . . . to note that among the other constituents of this idea . . . necessary existence is found . . . [and] . . . to maintain that it contains no contradiction. (Descartes 1952, p. 127)

In short, God is possible, according to Descartes, because our concept of God is clear and distinct. Or formally,

1 I (Descartes) have a clear and distinct idea of a supremely perfect being (God).
2 Whatever someone has a clear and distinct idea of possibly exists.
3 Therefore, a supremely perfect being possibly exists.

This is a valid argument for D3. But even if we grant the somewhat controversial assumption that clarity and distinctness are sufficient for logical possibility, the first premise might well be challenged as being all too subjective. Consider the comments of Leibniz:

> The reasoning of Descartes concerning the existence of the most perfect being assumed that the most perfect being can be known, or is possible.
>
> For this being assumed . . . it immediately follows that that being exists. But the question is asked whether it is within our power to conceive such a being . . . and [whether it] can be clearly known without contradiction. For the opponents will say that such a notion of the most perfect being . . . is a chimera. Nor is it sufficient for Descartes to appeal to experience and to allege that he perceives the same in such a manner in himself clearly and distinctly, for this is to break off, not complete the demonstration, unless he shows the method through which others also can attain the same experience . . . [otherwise] we wish to convince them by our authority alone. (1964, pp. 38–9)

Leibniz agrees with Descartes' ontological argument except for the subjective and authoritative argument for D3. Instead, Leibniz sketches an argument for God's possibility that is objective and *a priori* (1964, p. 38). Here is a valid reconstruction:

L1 All perfections are compatible.
L2 Every essential property of a supremely perfect being (God) is a perfection.
L3 If something's essential properties are perfections and all perfections are compatible, then its essential properties are compatible.
L4 If the essential properties of something are compatible, then it is possible that it exists.

Therefore,

D3 It is possible that a supremely perfect being (God) exists.

L2, L3, and L4 are self-evident. L1 is not. So Leibniz constructs an argument for L1 based on his definition of a *perfection* as a "simple quality which is positive and absolute, or expresses whatever it expresses without any limits" (1964, p. 37), and his belief that true propositions that express incompatibility are necessary truths which must be "either demonstrable or known *per se*" (1964, p. 38). A valid and arguably sound logical reconstruction of his argument is this:

S1 If any two perfections are compatible, then all perfections are compatible.
S2 If any two perfections are incompatible, then they are necessarily incompatible.
S3 If any two perfections are necessarily incompatible, then it is either self-evident that they are incompatible or it can be demonstrated that they are incompatible. (Because necessary truths are *a priori*, and *a priori* truths are either self-evident or demonstrable.)
S4 It is not self-evident that any two perfections are incompatible.
S5 If it can be demonstrated that any two perfections are incompatible, then either one is the negation of the other or some part of the one is incompatible with the other.
S6 If one perfection is the negation of the other, then one of them is not positive.
S7 Perfections are simple, positive qualities.
S8 If some part a perfection is incompatible with another, then one of them is not simple.

Therefore,

L1 All perfections are compatible.

2c. Critiques of the Descartes–Leibniz Ontological Argument

There are three fairly well-known critiques of the Descartes–Leibniz ontological argument. The first is Kant's claim that Descartes and Leibniz illicitly include *existence* as a property in the concept or essence of a supremely perfect being. According to Kant, existence is not a property at all. Even our logical reconstruction of the argument fails to avoid this critique by merely including the putative conditional property of *necessarily existing if at all* in the concept or essence of a supremely perfect being, for if existence is not a property, then it is not a property to exist necessarily if at all. Moreover, we explicitly assume that existence is a perfection in our Cartesian justification of D2.

Kant's main worry, if I read him correctly, is less about whether existence is a property per se, and more about whether it makes sense to include existence in the concept or essence of something. "'*Being*' is obviously not a real predicate; that is, it is not a concept of something which could be added to the concept of a thing" (1933, p. 504). Kant argues for this on the grounds that nothing new is said about anything that is said to exist.

> By whatever and by however many predicates we may think a thing . . . we do not make the least addition to the thing when we further declare that it *is*. Otherwise, it would not be exactly the same thing that exists, but something more than we had thought in the concept . . . (1933, p. 505)

Kant is half right and half wrong. He is right that existence is not a property in the usual sense of being includable in the concept of a thing. His explanation is that existence is not a property at all. A better explanation would be that we beg the question of the thing's very existence if we include existence in its concept or essence. True, we do not add to the *concept* of a thing when we say that it exists. So existential propositions are indeed synthetic. But, contrary to Kant, I think that we do predicate something new of a thing when we say that it exists. Kant seems to acknowledge as much when he says, "My financial position is affected . . . very differently by a hundred real thalers than it is by the mere concept of them" (1933, p. 505). Real money, because it exists, adds financial value to things that exist. Merely possible money, because it does not exist, does not add financial value to things that exist. It would seem, then, that Kant could consistently hold that existence is not only a property but also that it is a perfection, so long as it is not included in a thing's concept.[18]

The second critique is that the argument begs the question. But there is no evidence that it does because neither the Cartesian nor Leibnizian reasons for asserting the premises of the argument include the assertion that the conclusion is true. Perhaps the argument would beg the question were *existence* really included (by definition) in the concept of a supremely perfect being. However, our reconstruction of the argument does not make that assumption.

The third critique is that the argument is easily parodied. In the "First Set of Objections" to the *Meditations*, Caterus attempts to parody Descartes' ontological argument by saying that the same kind of argument could be used to prove the existence (in-reality) of an existent Lion, a being whose essence includes both being a lion and existing.

> This complex existent Lion includes both *lion* and the mode *existence* . . . *essentially* . . . But now, has not God from all eternity had a clear and distinct knowledge of this complex? . . . Yet . . . the distinct cognition of it which God possesses . . . does not constrain either part of the complex to exist, unless you assume that the complex does exist . . . Therefore . . . even though *you* have a distinct knowledge of the highest being, and granted that a being of supreme

18. Kant can still object to ontological arguments by arguing that a supremely perfect being is not an object of possible experience, and the synthetic proposition "a supremely perfect being exists" is neither a posteriori justifiable nor an *a priori* condition for the possibility of experience, the only two avenues of real knowledge for him.

> perfection includes *existence* in the concept of its essence, yet it does not follow that its existence is anything actual. . . (1952, p. 107)

Descartes parries this parody in "Reply to First Objections" by saying that:

> . . . even though other things are conceived only as existing, yet it does not thence follow that they do exist, but only that they may exist, because we do not conceive that there is any necessity for actual existence being conjoined with their other properties; but, because we understand that actual existence is necessarily and at all times linked to God's other attributes, it follows that God actually exists. (1952, p. 113)

What I think Descartes is struggling to say here is that an existent Lion is a contingent being, one that exists only if it possibly exists and possibly does not exist. This is because an existent Lion is still a lion, and lions are, by nature, contingent. All that relevantly follows from saying that *contingency* is contained in the concept or essence of an existent Lion is that anything that is an existent Lion possibly exists and possibly does not exist, if it exists. So if we assume that existent Lions possibly exist, we can only relevantly infer that it is possible that they possibly exist, and possible that they possibly do not exist. A supremely perfect being, by contrast, is a *necessary being*; one that exists noncontingently by nature. And, as we saw earlier, what follows from saying that the property of being a necessary being is contained in the concept or essence of a supremely perfect being is that a supremely perfect being exists necessarily if it exists at all. If we assume that a supremely perfect being possibly exists, we can validly infer that it does in fact exist. It appears, then, that Caterus' putative parody fails to refute Descartes' ontological argument.

Suppose, however, that we define a W* as a nonsupreme *necessary* being. We then get the parody "D1, K2, K3, K4/∴ KC" of Descartes' ontological argument, where K2, K3, K4, and KC are the same as D2, D3, D4, and DC, respectively, except for "W*" in place of "supremely perfect." Call this parody "the necessary being parody." Unlike Caterus' invalid putative parody of Descartes' argument, the necessary being parody is valid. But is it refuting?

Since the conclusion of the necessary being parody is obviously absurd, at least one of its premises must be false. So if each premise of the necessary being parody is at least as justifiable as the corresponding premise of Descartes' ontological argument, then the necessary being parody refutes it.

But it is not at all clear that each premise of the necessary being parody is at least as justifiable as the corresponding premise of Descartes' argument. Consider K3. We gave two arguments earlier for why D3 is true: Descartes' *clear and distinct* ideas argument, and Leibniz's *compatibility of perfections* argument. Might similar arguments be mustered in support of K3? I doubt it. An analogue of the Cartesian argument will not work because, I would guess, no one really has a clear and distinct idea of a W*. And a Leibnizian analogue will not work, because it must be false that every essential property of a W* (a nonsupreme being) is a perfection: otherwise, W* beings would be supremely perfect.

It is also doubtful that K4 could be justified in the same way that we justified D4 because it is not at all clear that W* is a perfection. And even if we were disposed to say that K4 is true because it is analytically or conceptually true that every of kind of thing is a thing of that kind essentially, then some other premise of the necessary being parody would have to be false. (It would also thereby make D4 analytically or conceptually true.) But both D1 and K2 are analytically true. So K3 would then definitely have to be false. Yet D3 is arguably true. Hence, the necessary being parody is nonrefuting.

3. Ontological Arguments of the Twentieth Century

Even though we made use of modal logic in proving the validity of our logical reconstruction of the ontological argument of Descartes and Leibniz, philosophers do not explicitly use modal reasoning in ontological arguments until the twentieth century. In this section, I shall present and very briefly discuss logical reconstructions of three explicitly modal arguments that are modeled on Anselm's ontological argument but do not assume that existence-in-reality is great-making. In the next section, I shall give an exposition and analysis of the lesser-known modal ontological argument of Gödel, which looks to be fashioned, in part, after Leibniz's proof of the compatibility of all perfections. And in the last two sections I shall present and briefly discuss two of my own ontological arguments: the modal perfection argument (MPA), and the temporal-contingency argument (TCA).

Norman Malcolm thought that Anselm had actually presented a convincing modal argument in *Proslogion III*. Here is a logical reconstruction of his 1960 rendition of that argument:

C1 It is possible that the greatest conceivable being exists.
C2 The greatest conceivable being is unlimited.
C3 Everything that is unlimited is so if and only if it does not depend on anything else for its existence or nonexistence and it neither just happens to exist nor just happens not to exist.
C4 Everything that does not depend on anything else for its existence or nonexistence is such if and only if no other being causes it to begin to exist and no other being causes it to cease to exist.
C5 Anything that begins to exist is caused to begin to exist by some other being, or it just happens to begin to exist.
C6 Anything that ceases to exist is caused to cease to exist by some other being, or it just happens to cease to exist.
C7 Anything that neither begins nor ceases to exist exists necessarily if it exists at all, and fails to exist necessarily if it exists at all.

Therefore,

C8 The greatest conceivable being exists.

Charles Hartshorne, who was also inspired by Anselm's *Proslogion III*, formulated a very elegant modal argument "1962, pp. 47–57". Here is a reconstruction:

H1 It is possible that a perfect being exists.
H2 Necessarily, if a perfect being exists, then a perfect being necessarily exists. (Anselm's Principle)

Therefore,

H3 A perfect being exists.

Then in 1974, Alvin Plantinga developed what he calls "A Victorious Modal Version" of the Anselmian argument that he embeds in the extensional language of possible worlds. A reconstruction is this:

P1 The property of being maximally great is exemplified in some possible world.
P2 The property of being maximally great is equivalent, by definition, to the property of being maximally excellent in every possible world.
P3 The property of being maximally excellent entails the properties of omniscience, omnipotence, and moral perfection.
P4 A universal property is one that is exemplified in every possible world or none.
P5 Any property that is equivalent to some property that holds in every possible world is a universal property.

Therefore,

P6 There exists a being that is essentially omniscient, omnipotent, and morally perfect (God).

These three modal arguments are valid.[19] But are they sound? A pretty good case might be made for C2–C6, H2, and P2–P5. But C7 is clearly questionable, especially if the modalities are construed logically, because an eternal being that does not exist in all possible words certainly is possible. And what should we think about the first premise of each of these arguments, which effectively says in each case that it is possible for God to exist? None of the authors of these respective arguments is particularly sanguine about proving this. Hartshorne merely suggests that we might "employ one or more of the other theistic proofs . . . to demonstrate that perfection must at least be conceivable" (1962, p. 52). Plantinga treats the possibility premise as a philosophical hypothesis, which he says it is rational to accept because otherwise "we should find ourselves with a pretty slim and pretty dull philosophy" (1974, p. 221). (Hardly the highest standard for what counts as rational!) And Malcolm says that he does "not know how to demonstrate the concept of God . . . is not self-contradictory" (1967, p. 318). Yet he assumes that it is not self-contradictory because it has "a place in the thinking and lives of human beings" (1967, p. 318).

One consequence of not attempting to prove that it is possible for God to exist is that the arguments of Malcolm, Hartshorne, and Plantinga do not beg the question. On the other hand, they thereby become particularly vulnerable to being refuted by parodies. For example, one might easily validly argue contra Hartshorne that Anselm's Principle, and the premise that it is possible that a supremely perfect being does not exist, jointly entail that a supremely perfect being does not exist. If we were merely to *postulate* that, possibly, a supremely perfect being exists, then we could also rightfully *postulate* that, possibly, a

19. See Appendix 2 for the deductions that prove that these are valid arguments.

supremely perfect does not exist. But then the parody would refute Hartshorne's argument because we should not be able to rightfully claim that the premises of the parody are less justifiable than those of Hartshorne's argument.

4a. Gödel's Ontological Argument

Gödel's "Ontological Proof" consisted of two very cryptic and highly technical handwritten pages dated February 10, 1970, which he subsequently shared with Dana Scott. It was first published as an appendix to Sobel's "Gödel's Ontological Proof" and then posthumously in the *Collected Works of Gödel*. Yet Gödel's ontological "proof" is still not widely known in the philosophical and theological communities, with only a dozen or so discussions of it in print, mostly by logicians, and is quite technical.

Gödel develops his ontological argument as a formal axiomatic theory with a theorem that says that there exists a so-called *God-like* being, where a being is God-like just in case it has every *positive* property. Although the second-order predicate "positive" is left undefined, Gödel suggests that it should be understood in either a moral-aesthetic sense (independent of the accidental structure of the world) or in the sense of pure attribution (as opposed to privation).[20] He cautions, however, not to interpret "positive" in the moral-aesthetic sense to mean the same thing as "good" (in the ordinary utilitarian sense) because "good" (in the ordinary utilitarian sense) means "greatest advantage + smallest disadvantage [which] is negative" (1995b, p. 435). Rather, he says that "positive" could be interpreted as "perfective," meaning "purely good" and implying nothing negative (1995b, p. 435). It is thus tempting to view the property of being positive in the moral-aesthetic sense as coextensive with the property of being a Platonic form. In other words, each Platonic form is positive, and each positive property is a Platonic form.

But "positive" in the sense of pure attribution rings more Leibnizian. In a footnote to his "Ontological Proof" Gödel says that a property (or proposition) that is expressed in "disjunctive normal form in terms of elementary properties [that] contains a member without negation" illustrates pure attribution (1995a, p. 404). And in his "Text" he says, "the positive properties are precisely those that can be formed out of the elementary ones through application of the operations &, V, $\supset$ " (1995b, p. 437). Adams interprets these cryptic remarks in Leibnizian fashion and suggests that "the purely positive properties will be those that involve no negation at all in their construction from elementary properties (provided the disjunction operation here too is inclusive)" (1995, p. 398).

Gödel's theory does not assume or presuppose that either existence or existence-in-reality is a property, and his logic has full existential import: the job of saying that something exists is performed by existential quantification. It has one primitive, three defined notions, five axioms, and three important theorems.[21]

Primitive

Property Y is *positive.*

$$P_1(Y) =_{df} \text{Property Y is positive}$$

20. Koons argues that the two conceptions of positivity "coincide perfectly" (2005, p. 3).

21. The formulation presented here is Scott's. But see Gödel's "Ontological Proof" (1995a, p. 403) or Sobel's "Gödel's Ontological Proof" (1987, pp. 256–7) for Gödel's own cryptic formulation.

Definitions

Df 1 A being has the property of being *God-like* (G_1) if and only if it has every positive property.

$$G_1 x =_{df} (Y)(P_1(Y) \supset Yx)$$

Df 2 A property is an *essence* (E_1) of something if and only if it has the property, and the property entails each of its properties.

$$E_1(Y, x) =_{df} Yx \,\&\, (Z)(Zx \supset \Box(y)(Yz \supset Zy))$$

Df 3 Something has the property of being a *necessary being* (N_2) if and only if every essence it has is necessarily instantiated.

$$N_2 x =_{df} (Y)(E_1(Y, x) \supset \Box(\exists z)Yz)$$

Axioms

Ax 1 A property is positive if and only if its negation is not positive.

$$(Z)(P_1(Z) \equiv \sim P_1(\hat{y}[\sim Zy]))$$

Ax 2 Positive properties entail only positive properties.

$$(Y)(Z)(P_1(Y) \,\&\, \Box(x)(Yx \supset Zx)) \supset P_1(Z))$$

Ax 3 God-likeness is positive.

$$P_1(G_1)$$

Ax 4 Positive properties are necessarily positive.

$$(Y)(P_1(Y) \supset \Box P_1(Y))$$

Ax 5 The property of being a necessary being is a positive.

$$P_1(N_2)$$

Theorems

Tm 1 It is possible that something is God-like.

$$\Diamond(\exists y)G_1 y$$

Tm 2 God-likeness is an essence of whatever is God-like.

$$(x)(G_1 x \supset E_1(G_1, x))$$

Tm 3 Something is God-like.

$$(\exists y)G_1 y$$

Proof-sketch of Tm 1.[22] Assume that it is not possible for something God-like to exist. Then God-likeness is an impossible property. Since impossible properties entail all properties, God-likeness entails the negation of God-likeness. Now God-likeness is positive by Ax 3. So the negation of God-likeness must be positive by Ax 2. But the negation of God-likeness cannot be positive by Ax 1. Therefore, by *reductio ad absurdum*, it must be possible that something is God-like.

22. I construct formal proofs of these three theorems in Appendix 2.

Proof sketch of Tm 2. Suppose that anything x is God-like, and that x has property Θ. Then Θ is positive: otherwise the negation of Θ would be positive by Ax 1; and if x had the negation of Θ, it would not have Θ. But then, Θ necessarily is positive by Ax 4. Now every positive property is possessed by anything that is God-like. So necessarily every property that is necessarily positive must be possessed by anything that is God-like. Since Θ is necessarily positive, it follows that necessarily Θ must be possessed by x. In other words, God-likeness entails Θ. Therefore, God-likeness is an essence of anything that is God-like.

Proof-sketch of Tm 3. Assume that something x is God-like. Since the property of being a necessary being is positive, and things that are God-like have all positive properties, x must be a necessary being. But things are necessary beings only if, for each of their essences, there is something that necessarily has that essence. Since God-likeness is an essence of x, it follows that something necessarily is God-like. Therefore, if it is possible that something is God-like, then it is possible that it is necessary that something is God-like. Now it is possible that something is God-like by Tm 1. So it is possible that it is necessary that something is God-like. But whatever is possibly necessary is necessary. And whatever is necessary is actually the case. Therefore, something is God-like.

4b. On Whether Gödel's Argument is Sound

Gödel's axioms imply that something is God-like.[23] While the argument is valid, Sobel shows that Gödel's axioms also imply the absurdity that every true proposition is necessarily true – the so-called modal collapse argument (1987, p. 253).[24] A modification of that argument shows that Gödel's argument cannot be sound:

> Assume that Gödel's argument is sound, and that it proves the existence of the God-like being g_1. By Tm 2 the property of being God-like is an essence of g_1. By the proof of Tm 3, God-likeness is necessarily instantiated. Let *p* be any contingent truth, and let Q be the property that a thing has if and only if *p*. It follows from the Principle of Abstraction that g_1 has Q if and only if *p*. This and the truth of *p* imply that g_1 has Q. Since the property of being God-like is an essence of g_1, the property of being God-like entails Q. But then property Q is necessarily instantiated, because the property of being God-like is necessarily instantiated. From Abstraction and Necessity Introduction, we have it that Q is necessarily instantiated if and only if it is necessarily the case that *p*. Therefore, *p* is necessarily true, which contradicts our assumption that *p* is contingent. So Gödel's argument cannot be sound.

At least one of Gödel's axioms must be false. Sobel objects to Ax 2, Ax 3, and Ax 5. His argument against Ax 2 is this:

23. Gödel's five axioms are the premises of his ontological argument, and Tm 3 is the conclusion.
24. Sobel also shows that Gödel's axioms imply that if everything has an essence, $(x)(\exists Y)E_1(Y, x))$, then everything is a necessary being, $(x)N_2x$. But that result does not prove that Gödel's argument is unsound. It makes perfect sense to think that free or contingent things do not have essences in Gödel's sense of essence. Ironically, Sobel notes that things with no essence are necessary beings: $(x)(\sim(\exists Y)(E_1(Y, x) \supset ((Y)(E_1(Y, x) \supset \Box(\exists z)Yz))$. That is true, vacuously; and there is no warrant thereby for claiming that everything is a necessary being, or that some essence of a free being is necessarily instantiated (1987, p. 252).

> Gödel's generous interpretation of properties is at least awkward for an axiological interpretation of Axiom 2, since, according to it, if there is a positive property, then every necessarily universal property such as being self-identical, and being either red or not red, is a positive property. (2004, p. 120)

There are two quick challenges to this argument. First, Gödel's argument for a God-like being is valid even if tautological or necessarily universal properties are positive. Second, if Ax 2 were replaced by a modification which says that nontautological properties which are entailed by a positive property are positive, and Ax 3 were replaced by a modification which says that *God-likeness* is nontautological and positive, then the resulting ontological argument would still be valid, but its premises would not imply that tautological properties are positive.

Hàjek argues that there are difficulties with either Ax 2 or Ax 3. Let Devil-likeness (D_1) be the property of having all properties that are not positive. Now God-likeness entails the property of being either God-like or Devil-like. So by Ax 2 and Ax 3, the property of being either God-like or Devil-like ($G_1 \vee D_1$) must be positive. Hájek believes that that result is "counterintuitive." Maybe it is and maybe it is not; but even if it is counterintuitive, that does not mean that either Ax 2 or Ax 3 is false.

Sobel says that the difficulty is worse than counterintuitive. "There is *prima facie* no more reason for saying that that [a] disjunctive property is positive than there is for saying that it is not positive; it is entailed by a property that is positive according to Axiom 3 . . . and it is entailed by a property that is 'equally negative'" (2004, p. 122). True enough, it is reasonable to assume that D_1 is negative (not positive), and D_1 surely entails ($G_1 \vee D_1$). But Sobel is surely wrong to presuppose that these two things imply that ($G_1 \vee D_1$) is negative. Properties that are negative entail properties that are positive, but not vice versa. The property of being morally evil, for example, entails the property of having some intelligence.

Ax 5 is the axiom that Sobel objects to the most. He maintains that nothing worthy of worship can exist necessarily. And he argues that if the property of being a necessary being is positive, and if any being whose essence is to be worthy of worship can be assumed have to have all positive properties, which is plausible, then any being worthy of worship would have to have the property of being a necessary being and would, therefore, exist necessarily if all.[25] So if we abandon Ax 5, we not only block the proof of Tm 3, as well as the modal collapse argument for why Gödel's argument is not sound, we also mollify Sobel's angst about objects of worship being necessary beings. I shall return later to the issue of Ax 5.

25. Sobel holds that Gödel's god (or the *demonstrable* god of any ontological argument) cannot be the God of Theism (1987, pp. 254–5). When fully stated in standard logical form, his argument appears to be this:

1 Gödel's god is a necessary being.
2 Every necessary being is an abstract entity.
3 No abstract entity is worthy of being worshipped.
4 The God of Theism is worthy of being worshipped.
5 Therefore, Gödel's god is not the God of Theism

Although valid, the second premise of this argument is arguably false. I see no good reason for not thinking that something with causal powers could exist in every possible world. But it is hard to imagine an abstract entity with causal powers. So some necessary beings might not be abstract entities.

What about Ax 1? Anderson splits Ax 1 into Ax 1^a and Ax 1^b:

Ax 1^a If a property is positive, then its negation is not positive

$$(Z)(P_1(Z) \supset \sim P_1(\hat{y}[\sim Zy]))$$

Ax 1^b If the negation of a property is not positive, then the property is positive.

$$(Z)(\sim P_1(\sim \hat{y}[\sim Z] \supset P_1(Z)))$$

He then persuasively argues that Ax 1^a is true, and he correctly notes that Ax 1^b is false.[26] Many properties and their negations appear not to be positive, such as the property of being red and the property of not being red. But if we abandon Ax 1 in favor of Ax 1^a, and make no other changes, we also block the proof of Tm 3. Interestingly, Ax 1^b is also used in our version of the Sobel modal collapse argument, because an instance of Ax 1^b occurs in line 3 of the proof of Tm 2.

Anderson, Hazen, Koons, and Hàjek have all formulated different emendations of Gödel's theory that successfully dodge Sobel's modal collapse refutation of Gödel's ontological argument. Since space here will not permit a detailed analysis, comparison, and evaluation of all four emendations, I shall provide just a summary of the key ideas of Anderson's emendation. I shall then briefly discuss a class of ingenious parodies designed by Graham Oppy that he says inflict Anderson's theory, and quite possibly the others, too.

Anderson's emendation has three new definitions, Df 1^a, Df 2^a, and Df 3^a. Its axioms are Ax 1^a, Ax 2, Ax 3^a, Ax 4, and Ax 5^a.

Df 1^a A being has the property of being *God^a-like* ($G^a{}_1$) if and only if its essential properties are all and only those properties that are positive.[27]

Df 2^a A property Y is an *essence^a* ($E^a{}_1$) of being x if and only if, for every property Z, x has Z essentially if and only if Y entails Z.

Df 3^a Something has the property of being a *necessary^a being* ($N^a{}_2$) if and only if every essence^a it has is necessarily instantiated.

Ax 3^a God^a-likeness is positive.

Ax 5^a The property of being a necessary^a being is positive.

Anderson's axioms imply that something is God^a-like (Anderson's ontological argument). But Sobel's modal collapse argument is not valid in Anderson's theory.[28] One very interesting feature of Anderson's theory, unlike Gödel's, is that it allows for the possibility that God^a-like beings have some nonpositive properties contingently. Another is that the concept of *essence^a* closely resembles the common philosophical concept of *essence*, whereas an essence for Gödel is best understood as a complete characterization.

Oppy's theory (1996) is just like Anderson's except for Df 1* in place of Df 1^a, Ax 3* in place of Ax 3^a, and Ax 5* in place of Ax 5^a.

26. Anderson deduces Ax 1^a from *plausible* principles about intrinsic preferability (1990, p. 295).

27. Although Anderson includes a "*" in the names of his key terms and axioms to distinguish them from those of Gödel, we instead include an "a" in the names of Anderson's key terms and axioms in order to distinguish them from names used by Oppy, who also includes a "*" in the names of still other Gödel-like terms and axioms.

28. Our modification of Sobel's argument shows why. The property Q that a thing has if and only if *p* (where *p* is some contingent truth) cannot be an essential property of a God^a-like being, and God^a-likeness does not entail Q.

Df 1* A being has the property of being *God*-like* (G^*_1) if and only if its essential properties are those and only those which are positive, except for $\Phi_1, \ldots, \Phi_n$.

Ax 3* God*-likeness is positive.

Ax 5* Necessary existence is a positive property, and distinct from $\Phi_1, \ldots, \Phi_n$.

His axioms imply the absurdity that there are almost as many God*-like beings as there are positive properties (Oppy's ontological parody).[29]

Oppy misinterprets his ontological parody as showing that at least one of Anderson's axioms must be false, and he only conjectures that an "obvious candidate is [Ax 3^a]" (2000, p. 2).[30] It is also certain that not all of Oppy's axioms can be true because the conclusion of his valid parody is absurd. Oppy's parody will, however, substantively refute Anderson's ontological argument if and only if Ax 3* and Ax 5* are at least as justifiable as Ax 3^a and Ax 5^a, respectively, assuming that Ax 1^a, Ax 2, and Ax 4 are at least modestly justifiable (more so than not); and Oppy's parody will vacuously refute Anderson's ontological argument if either Ax 1^a, Ax 2, or Ax 4 is not even modestly justifiable.

We could show that Oppy's parody does not substantively refute Anderson's ontological argument if we could assume that the property of not having Φ_i essentially is not positive, given that Φ_i is a positive property excluded by definition from being an essential property of anything that is God*-like. For we could then prove that Ax 3* is false with the following Gettings (1999)-style argument:

1 The property of being God*-like entails the property of not having Φ_i essentially. (Derived from the definiens of "God*-like.")[31]
2 If the property of being God*-like is positive, and the property of being God*-like entails the property of not having Φ_i essentially, then the property of not having Φ_i essentially is positive. (Ax 2)
3 The property of not having Φ_i essentially is not positive.
∴ The property of being God*-like is not positive.

It would be difficult to know, however, whether Oppy's parody substantively refutes Anderson's argument without knowing more about what a positive property is, and whether it is true that the property of not having Φ_i essentially is not positive.[32] In the next section,

29. Oppy constructs a slightly different parody of a slight modification of Anderson's argument that includes the new premise that if a property Z is positive then the property $\hat{y}[\Box Zy]$ is positive (2007, pp. 16–7).

30. Instead of arguing directly for the falsity of Ax 3^a, Oppy merely says, "atheists and agnostics may (perhaps should) say that the property of being [God^a-like] is positive only if it is exemplified," and then notes that if this new proposition were to replace Ax 3^a of Anderson's argument, then the resulting argument would beg the question (2000, p. 2). This is a red herring, and it fails to show that Ax 3^a itself is false.

31. Symbolically: $\Box(x)[(Y)((P_1(Y) \,\&\, Y \neq \Phi_i) \equiv \Box Yx) \supset \sim\Box\Phi_i x]$. Use a conditional proof and utilize the fact that "$\Phi_i = \Phi_i$" is a logical truth.

32. If the mere presence of the word "not" in a description of a property were a guarantor of nonpositivity, then the property of not having property Φ_i essentially would be nonpositive. But the presence of "not" does not guarantee nonpositivity. Otherwise, the property of not having the negation of a positive property would be nonpositive, which it is not.

We might attempt to prove that the property of not having Φ_i essentially is not positive with another Gettings (1999)-style argument:

a.	$P_1(\Phi_i)$	given
b.	$(Y)(P_1(Y) \supset P_1(\hat{y}[\Box Yy]))$	new axiom
c.	$P_1(\Phi_i) \supset \sim P_1(\hat{y}[\sim\Box\Phi_i y])$	Ax 1^a
∴	$\sim P_1(\hat{y}[\sim\Box\Phi_i y])$	

The problem is that the "new axiom" is no more obvious than the conclusion.

I shall replace the predicate "is a positive property" with the predicate "is a perfection," and I shall replace "is God^a-like" with "is supreme." I shall then argue that the analogues of Ax 1^a, Ax 2, and Ax 3^a are true, but that the analogue of Ax 3* is false. Those three analogues will constitute the premises of the MPA. The analogues of Ax 4 and Ax 5^a will not be needed.

5. The Modal Perfection Argument

The Modal Perfection Argument (MPA) is an ontological argument that is rooted in the ontological arguments of Anselm, Descartes–Leibniz, and Gödel.[33] Think of a perfection (P_2) as a property that it is necessarily better to have than not; and define the property of being supreme (S_1) as the property that a thing has if and only if it is impossible for something to be greater and impossible for there to be something else than which it is not greater: $S_1x =_{df} (\sim\Diamond(\exists y)Gyx \;\&\; \sim\Diamond(\exists y)(x\neq y \;\&\; \sim Gxy))$. The conclusion of MPA is that exactly one supreme being exists,[34] and the premises are the following:

M1 A property is a perfection only if its negation is not a perfection.
M2 Perfections entail only perfections.
M3 The property of being supreme is a perfection.

We can show that MPA is valid by first showing that M1, M2, and M3 jointly imply that it is possible that a supreme being exists. The proof is the same as our proof of Gödel's Tm 1 in Appendix 2, save for "P_2" in place of "P_1" and "S_1" in place of "G_1." (The annotation also substitutes M1, M2, and M3 for A1, A2, and A3, respectively, and drops "Equiv" and "Simp" from line 10.) We can then prove that the possibility of a supreme being implies the existence of a supreme being as follows:

Deduction

1	$\Diamond(\exists x)S_1x$	pr
2	$\Diamond(\exists x)S_1x \supset (\exists x)\Diamond S_1x$	theorem[35]
3	$(\exists x)\Diamond S_1x$	1, 2 MP
4	$\Diamond S_1\nu$	3, EI
5	$\Diamond(\sim\Diamond(\exists y)Gy\nu \;\&\; \sim\Diamond(\exists y)(\nu\neq y \;\&\; \sim G\nu y))$	4, df "S_1"
6	$\Diamond(\sim\Diamond(\exists y)Gy\nu \;\&\; \sim\Diamond(\exists y)(\nu\neq y \;\&\; \sim G\nu y)) \supset$ $(\Diamond\sim\Diamond(\exists y)Gy\nu \;\&\; \Diamond\sim\Diamond(\exists y)(\nu\neq y \;\&\; \sim G\nu y))$	theorem

33. I first formulated a version of MPA in November 2001, and I presented it at The Second Annual Saint Anselm Conference held in April 2002 at Saint Anselm College, Manchester, NH. *Philo* published an improved version of MPA by Maydole (2003), replies by Oppy (2004) and Metcalf (2005), and my counterreplies (2005a and 2005b).
34. MPA does not assume or presuppose that existence is a property, and its quantifiers have existential import.
35. "$\Diamond(\exists x)S_1x \supset (\exists x)\Diamond S_1x$" is an instance of the controversial Barcan Formula (BF). Plantinga argues against BF (1974, pp. 59–60). I refute his argument (Maydole 1980, pp. 140–2). And I argue for BF (Maydole 2003, pp. 303–7).

7	$(\Diamond\sim\Diamond(\exists y)Gy\nu \ \& \ \Diamond\sim\Diamond(\exists y)(\nu\neq y \ \& \sim G\nu y))$	4, 5 MP
8	$\Diamond\sim\Diamond(\exists y)Gy\nu$	7 Simp
9	$\Diamond\sim\Diamond(\exists y)(\nu\neq y \ \& \sim G\nu y)$	7 Com, Simp
10	$\Diamond\sim\Diamond(\exists y)Gy\nu \supset \sim\Diamond(\exists y)Gy\nu$	theorem
11	$\Diamond\sim\Diamond(\exists y)(\nu\neq y \ \& \sim G\nu y) \supset \sim\Diamond(\exists y)(\nu\neq y \ \& \sim G\nu y)$	theorem
12	$\sim\Diamond(\exists y)Gy\nu$	8, 10 MP
13	$\sim\Diamond(\exists y)(\nu\neq y \ \& \sim G\nu y)$	9, 11 MP
14	$\sim\Diamond(\exists y)Gy\nu \ \& \sim\Diamond(\exists y)(\nu\neq y \ \& \sim G\nu y)$	12, 13 Conj
15	$S_1\nu$	14, df "S_1"
16	$(\exists x)S_1x$	15 EG

It is also logically true that, *at most*, one thing is supreme. (Maydole 2003, p. 302). Therefore, exactly one supreme being exists.

I show that M1, M2, and M3 are true, and that MPA is resistant to sundry salient parodies (Maydole 2003). Here is a small snapshot of how I argue there for the premises: M1 is true because it is better to have a property than not only if it is not better to not have that property than not; M2 is true because it is always better to have that which is a necessary condition for whatever it is better to have than not; and M3 is true because it is reasonable to assume that a thing is supreme if and only if it is necessarily greater than everything else solely by virtue of having some set of perfections, making the extension of the property of being supreme identical with the intersection of the extensions of those perfections. Premises of a different argument for the truth of M3 might be as follows:

M31 For every Z, all of the nontautological essential properties entailed by Z are perfections if and only if the property of being a Z is a perfection.

M32 Every nontautological essential property entailed by the property of being supreme is a perfection.

Therefore,

M3 The property of being supreme is a perfection.

This argument is clearly valid, and the premises are plausible. M31 is arguably true because (1) it is necessarily better to have a property if and only if the property endows whatever has it with nontautological properties that are necessarily better to have than not, and (2) for any properties Y and Z, if Z endows something with Y, then Z entails Y, and (3) perfections are properties that are better to have than not. M32 is arguably true because (4) all the nontautological essential properties entailed by the essence of a supreme being are perfections, and (5) anything entailed by the essence of a thing of kind Z is entailed by the property of being a Z.

There is no evidence to indicate that MPA begs the question. And we can easily show that an Oppy-style parody based on the idea of being almost supreme does not refute MPA, where something is almost supreme just in case it is impossible for anything to be almost greater: from M31, and the fact that it is not the case that every nontautological essential property entailed by the property of being almost supreme is a perfection, it follows that the property of being almost supreme is not a perfection.

6. The Temporal-Contingency Argument

We have shown that the first three premises of MPA imply that it is possible that a supreme (greatest possible) being exists. But any valid argument for the existence of a supreme being that has copossible premises would show the same thing, even if one or more of its premises happens to be false. Consider the Third Way of St Thomas Aquinas.

> The third way is taken from possibility and necessity and runs thus. We find in nature things that are possible to be and not possible to be, since they are found to be generated and corrupted. But it is impossible for these always to exist, for that which can not-be at some time is not. Therefore, if everything can not-be, then at one time there was nothing in existence. Now if this were true then even now there would be nothing in existence, because that which does not exist begins to exist only through something already existing. Therefore if at one time nothing was in existence, it would have been impossible for anything to have begun to exist; and thus now nothing would be in existence – which is absurd. Therefore, not all beings are merely possible, but there must exist something the existence of which is necessary. But every necessary thing has its necessity caused by another, or not. Now it is impossible to go on to infinity in necessary things which have their necessity caused by another, as has already been proved in regard to efficient causes. Therefore, we cannot but admit the existence of some being having of itself its own necessity, and not receiving it from another, but rather causing in others their necessity. This all men speak of as God. (1998, pp. 4–5)

Aquinas' Third Way is invalid *per se* because the proposition that everything fails to exist at some time does not entail the proposition that there is a time when everything fails to exist. Temporally contingent things might be eternal in the actual world yet fail to exist at some time in some other possible world. But the Third Way is fertile, and it can easily be transformed into a valid argument for the existence of a supreme being that has copossible premises.

Think of something as *generated* just in case there is a time when it exists and an earlier time when it does not; and as *corrupted* just in case there is a time when it exists and a later time when it does not. Define something as *temporally necessary* if and only if it is necessarily not generated and necessarily not corrupted. And define something as *temporally contingent* if and only if it is possibly generated or possibly corrupted. Then our modified Third Way is this.

T1 Something presently exists.
T2 Only finitely many things have existed to date.
T3 Every temporally contingent being begins to exist at some time and ceases to exist at some time.
T4 Everything that begins to exist at some time and ceases to exist at some time exists for a finite period of time.
T5 If everything exists for only a finite period of time, and there have been only finitely many beings to date, then there was a time when nothing existed.
T6 If there was a time when nothing existed, then nothing presently exists.
T7 A being is temporally necessary if and only if it is not temporally contingent.
T8 Everything has a sufficient reason for its existence.
T9 Anything that has a sufficient reason for its existence also has a sufficient reason for its existence that is a sufficient reason for its own existence.

T10 No temporally contingent being is a sufficient reason for the existence of a temporally necessary being.
T11 Every temporally necessary being that is a sufficient reason for its own existence is a being without any limitations.
T12 A being without any limitations is necessarily greater than any other being.
T13 It is not possible for anything to be greater than itself.
T14 It is necessarily the case that "greater than" is asymmetric.

Therefore,

T15 There exists a supreme being.

The Temporal-Contingency Argument (TCA) is the argument "$\Diamond$(T1 & T2 & . . . & T14) /∴ $(\exists x)S_1x$." The later deduction for "(T1, T2, . . . , T14) /∴ $(\exists x)S_1x$" plus the deduction (in the previous section) for "$\Diamond(\exists x)S_1x$ /∴ $(\exists x)S_1x$" together prove that TCA is valid.[36] First, let us add to our lexicon, thus

$B_2x =_{df}$ x begins to exist at some time and ceases to exist at some time
$T_2x =_{df}$ x is temporally necessary
$C_2x =_{df}$ x is temporally-contingent
$F_2x =_{df}$ x exists for a finite period of time
$M_2 =_{df}$ Only finitely many things have existed to date
$P_3 =_{df}$ Something presently exists
$N_2 =_{df}$ There was a time when nothing existed
$S_2xy =_{df}$ x is a sufficient reason for y for the existence of y
$W_2x =_{df}$ x is without any limitations

Deduction[37]

1	P_3	pr 1
2	M_2	pr 2
3	$(x)(C_2x \supset B_2x)$	pr 3
4	$(x)(B_2x \supset F_2x)$	pr 4
5	$((x)F_2x \ \& \ M_2) \supset N_2$	pr 5
6	$N_2 \supset \sim P_3$	pr 6
7	$(x)(T_2x \equiv \sim C_2x)$	pr 7
8	$(x)C_2x$	AIP
9	$C_2\mu \supset B_2\mu$	3 UI
10	$C_2\mu$	8 UI
11	$B_2\mu$	9, 11 MP

36. The modified Third Way that I present here is a variation on a modification of Aquinas' Third Way that I discuss in "A Modal Model . . ." (1980, pp. 139–40). It is also different from the central argument (MTW) of my "The Modal Third Way" (2000, pp. 1–28). MTW is sound, but the supreme being it proves is not defined as a greatest possible being; and it is not necessarily the God of Anselm, unless we postulate that such a supreme being is a greatest possible being. By contrast, the supreme being of TCA is a greatest possible being by definition.
37. This deduction departs slightly from the norm of beginning with all premises.

12	$B_2u \supset F_2u$	4 UI
13	F_2u	11,12 MP
14	$(x)F_2x$	13 UG
15	$(x)F_2x \mathbin{\&} M_2$	2, 14 Conj
16	N_2	5, 15 MP
17	$\sim P_3$	6, 16 MP
18	$P_3 \mathbin{\&} \sim P_3$	1, 17 Conj
19	$\sim(x)C_2x$	8–19 IP
20	$(\exists x)\sim C_2x$	19 QN
21	$\sim C_2\nu_1$	20 EI
22	$T_2\nu_1 \equiv \sim C_2\nu_1$	7 UI
23	$(T_2\nu_1 \supset \sim C_2\nu_1) \mathbin{\&} (\sim C_2\nu_1 \supset T_2\nu_1)$	22 Equiv
24	$(\sim C_2\nu_1 \supset T_2\nu_1)$	23 Com, Simp
25	$T_2\nu_1$	21, 24 MP
26	$(\exists x)T_2x$	25 EG
27	$(x)(\exists y)S_2yx$	pr 8
28	$(x)[(\exists y)S_2yx \supset (\exists z)(S_2zx \mathbin{\&} S_2zz)]$	pr 9
29	$(x)(y)[(T_2x \mathbin{\&} S_2yx) \supset \sim C_2y]$	pr 10
30	$(y)[(T_2y \mathbin{\&} S_2yy) \supset W_2y]$	pr 11
31	$(y)[W_2y \supset \Box(z)(z \neq y \supset Gyz)]$	pr 12
32	$\sim\Diamond(\exists y)Gyy$	pr 13
33	$\Box(x)(y)(Gxy \supset \sim Gyx)$	pr 14
34	$(\exists y)S_2y\nu_1$	27, UI
35	$(\exists y)S_2y\nu_1 \supset (\exists z)(S_2z\nu_1 \mathbin{\&} S_2zz)$	28, UI
36	$(\exists y)(S_2z\nu_1 \mathbin{\&} S_2zz)$	34, 35 MP
37	$S_2\nu\nu_1 \mathbin{\&} S_2\nu\nu$	36, EI
38	$(T_2\nu_1 \mathbin{\&} S_2\nu\nu_1) \supset \sim C_2\nu$	29, UI twice
39	$S_2\nu\nu_1$	37 Simp
40	$T_2\nu_1 \mathbin{\&} S_2\nu\nu_1$	25, 39 Conj
41	$\sim C_2\nu$	38, 40 MP
42	$T_2\nu \equiv \sim C_2\nu$	7, UI
43	$(T_2\nu \supset \sim C_2\nu) \mathbin{\&} (\sim C_2\nu \supset T_2\nu)$	42 Equiv
44	$\sim C_2\nu \supset T_2\nu$	43 Com, Simp
45	$T_2\nu$	44, 41 MP
46	$S_2\nu\nu$	37 Com, Simp
47	$T_2\nu \mathbin{\&} S_2\nu\nu$	45, 46 Conj
48	$(T_2\nu \mathbin{\&} S_2\nu\nu) \supset W_2\nu$	30 UI
49	$W_2\nu \supset \Box(z)(z \neq \nu \supset G\nu z)$	31 UI
50	$\Box(z)(z \neq \nu \supset G\nu z)$	47, 48, 49 MP
51	$\Box(z)(\sim z \neq \nu \vee G\nu z)$	50 Impl
52	$\Box(z)(\sim z \neq \nu \vee \sim\sim G\nu z)$	51 DN
53	$\Box(z)\sim(z \neq \nu \mathbin{\&} \sim G\nu z)$	52 DeM
54	$\Box\sim(\exists z)(z \neq \nu \mathbin{\&} \sim G\nu z)$	53 QN
55	$\sim\Diamond(\exists z)(z \neq \nu \mathbin{\&} \sim G\nu z)$	54 ME
56	$\Box\sim(\exists y)Gyy$	32 ME
57	$\Box(y)\sim Gyy$	56 QN
58	$(y)\sim Gyy$	ACP

59	$\sim G\mu\mu$	58 UI
60	$\sim G\mu\mu \vee \nu\neq\mu$	59 Add
61	$\nu\neq\mu \vee \sim G\mu\mu$	60 Com
62	$\nu=\mu \supset \sim G\mu\mu$	61 Impl
63	$\mu=\nu \supset \nu=\mu$	theorem
64	$\mu=\nu \supset \sim G\mu\nu$	62, 63 HS
65	$(y)\sim Gyy \supset (\mu=\nu \supset \sim G\mu\nu)$	58–64 CP
66	$\Box[(y)\sim Gyy \supset (\mu=\nu \supset \sim G\mu\nu)]$	65 (theorem) NI
67	$\Box(\mu=\nu \supset \sim Gu\nu)$	57, 66 MMP
68	$\Box(x)(y)(Gxy \supset \sim Gyx) \;\&\; \Box(z)(z\neq\nu \supset G\nu z)$	33, 50 Conj
69	$\Box\,[(x)(y)(Gxy \supset \sim Gyx) \;\&\; \Box(z)(z\neq\nu \supset G\nu z)] \supset$ $\Box[(x)(y)(Gxy \supset \sim Gyx) \;\&\; (z)(z\neq\nu \supset G\nu z)]$	theorem
70	$\Box[(x)(y)(Gxy \supset \sim Gyx) \;\&\; (z)(z\neq\nu \supset G\nu z)]$	68, 69 MP
71	$\Box\,\{[(x)(y)(Gxy \supset \sim Gyx) \;\&\; (z)(z\neq\nu \supset G\nu z)] \supset$ $(\mu\neq\nu \supset \sim G\mu\nu)\}$	theorem
72	$\Box(\mu\neq\nu \supset \sim G\mu\nu)$	70, 71 MMP
73	$[\Box(\mu=\nu \supset \sim G\mu\nu) \;\&\; \Box(\mu\neq\nu \supset \sim G\mu\nu)] \supset$ $[\Box(\mu=\nu \vee \mu\neq\nu) \supset \Box(\sim G\mu\nu \vee \sim G\mu\nu)]$	theorem
74	$\Box(\mu=\nu \supset \sim Gu\nu) \;\&\; \Box(\mu\neq\nu \supset \sim G\mu\nu)$	67, 72 Conj
75	$\Box[(\mu=\nu \vee \mu\neq\nu) \supset (\sim G\mu\nu \vee \sim G\mu\nu)]$	73, 74 MP
76	$\Box(\mu=\nu \vee \mu\neq\nu)$	theorem
77	$\Box(\sim G\mu\nu \vee \sim G\mu\nu)$	75, 76 MMP
78	$\Box(\sim G\mu\nu \vee \sim G\mu\nu) \supset \Box\sim G\mu\nu$	theorem
79	$\Box\sim G\mu\nu$	77, 78 MP
80	$(z)\Box\sim Gz\nu$	79, UG
81	$(z)\Box\sim Gz\nu \supset \Box(z)\sim Gz\nu$	theorem[38]
82	$\Box(z)\sim Gz\nu$	80, 81 MP
83	$\Box\sim(\exists z)Gz\nu$	82, QN
84	$\sim\Diamond(\exists z)Gz\nu$	83, ME
85	$\sim\Diamond(\exists z)Gz\nu \;\&\; \sim\Diamond(\exists z)(z\neq\nu \;\&\; \sim G\nu z)$	84, 55 Conj
86	$S_1\nu$	85, def "S_1"
87	$(\exists x)S_1x$	86 EG

The sole premise of TCA is true if and only if there is some possible world where the premises T1, T2 . . . and T14 are true. Now I know of no reason to believe that there could not be a possible world ω where the propositions T1, T2, T3, T6, T8, T9, T10, and T11 express logically contingent facts about ω. Propositions T4, T5, T7, T13, and T14 appear to be self-evident analytic truths which are true in every possible world, including ω. Only T12 requires special justification:

> Assume x is a being without limitations in ω. Then x possesses every great making property in ω. In particular, x possesses the property in ω of not being limited in world ω_1 by anything.

38. Line 81 is an instance of another version of the BF. The two versions are equivalent, by the rules, to Trans, DN, ME, and QN. It is also interesting to note, however, that the validity of our modified Third Way can be proven in the logic QS4, which is just like QS5, save for the rule $\ulcorner\Diamond p \;/\therefore\; \Diamond\Diamond p\urcorner$ in place of $\ulcorner\Diamond p \;/\therefore\; \Box\Diamond p\urcorner$. The BF is not a theorem of QS4.

> In other words, if x is a being without any limitations in ω, then x possesses every great making property in ω. But the property of not being limited in ω_1 is a great making property of ω. So it is true in ω that it is true in ω_1 that x is unlimited. But for any statement *p*, if it is true in world α that *p* is true in world β, then *p* is true in world β. Hence, x is unlimited in world ω_1. Now if x is unlimited in ω_1, then in ω_1 x is greater than any other being in ω_1; otherwise x would be limited by not possessing a great making property possessed by something else. Hence it is true in ω_1 that x is greater than every other being. Since ω_1 is an arbitrarily selected possible world, it follows that it is true in every possible world that x is greater than every other being. Consequently, it is necessarily the case that x is greater than every other being. So T12 is true in ω. (Maydole 1980, p. 140)

TCA is a quasi-ontological argument that is arguably sound. There is also no evidence to indicate that it begs the question. And it seems that it would be particularly resistant to being parodied, given its dependence on sundry logically contingent facts about a possible world, and the historical absence of any parodies against Third Way arguments.

7. Conclusion

Ontological arguments are captivating. They convince some people but not others. Our purpose here was not to convince but simply to show that some ontological arguments are sound, do not beg the question, and are insulated from extant parodies. Yet good logic does convince sometimes. Other times, something else is needed.

References

Adams, R. M. (1994) *Leibniz: Determinist, Theist, Idealist*. New York: Oxford University Press.

Adams, R. M. (1995) Introductory note to *1970. In S. Feferman (ed.), *Kurt Gödel, Collected Works*, vol lll, 388–402. New York: Oxford University Press.

Anderson, C. A. (1990) Some emendations of Gödel's ontological proof. *Faith and Philosophy* 7, 291–303.

Anselm (1962) *Basic Writing*, 2nd edn. Trans. S. N. Deane. La Salle, IL: Open Court.

Anselm (1967) Proslogion (*Chapters II–IV*). Trans. A. C. McGill. In J. H. Hick and A. C. McGill (eds.), *The Many-Faced Argument*, 3–8. New York: Macmillan.

Anselm (1998) Proslogion. Trans. M. J. Charlesworth. In B. Davies and G. Evans (eds.), *Anselm of Canterbury: The Major Works*, 1–34. New York: Oxford University Press.

Thomas Aquinas (1998) *Summa Theologica*, First Part, Q. II, Art. 3. Trans. A. C. Pegis. Reprinted in *Philosophy of Religion*, 3rd edn. Ed. L. P. Pojman. Belmont, CA: Wadsworth.

Copleston, F. C. (1961) *Medieval Philosophy*. New York: Harper Torchbooks.

Descartes, R. (1952) *Meditations on First Philosophy* and *Objections Against the Meditations and Replies* (including "arguments demonstrating the existence of God . . . drawn up in geometrical fashion"). Trans. E. S. Haldane and G. R. T. Ross. In R. M. Hutchins (ed.), *The Great Books*, vol. 31, 69–293. Chicago: Encyclopedia Britannica.

Findlay, J. N. (1998) Can God's existence be disproved? In L. P. Pojman (ed.), *Philosophy of Religion*, 3rd edn, 93–7. Belmont, CA: Wadsworth.

Gettings, M. (1999) Gödel's ontological argument: a reply to Oppy. *Analysis* 59: 4, 309–13.

Girle, R. (2003) *Possible Worlds*. Montreal: McGill-Queens University Press.

Gödel, K. (1995a) Ontological proof (*1970). In S. Feferman (ed.), *Kurt Gödel, Collected Works*, vol. III, 403–4. New York: Oxford University Press.

Gödel, K. (1995b) Texts relating to the ontological proof. In S. Feferman (ed.), *Kurt Gödel, Collected Works*, vol. III, 429–37. New York: Oxford University Press.

Gustason, W. and Ulrich, D. E. (1989) *Elementary Symbolic Logic*. Prospect Heights, IL: Waveland Press.

Hàjek, P. (2002) A new small emendation of Gödel's ontological proof. *Studia Logica* 71, 149–64.

Hartshorne, C. (1962) *The Logic of Perfection and Other Essays in Neoclassical Metaphysics*. La Salle, IL: Open Court.

Hazen, A. P. (1998) On Gödel's ontological proof. *Australian Journal of Philosophy* 76, 361–77.

Kant, I. (1933) *Critique of Pure Reason*. Trans. N. K. Smith. New York: St. Martin's Press.

Konyndyk, K. (1986) *Introductory Modal Logic*. Notre Dame, IN: University of Notre Dame Press.

Koons, R. C. (2005) Sobel on Gödel's Ontological Proof, http://www.scar.utoronto.ca/~sobel/OnL_T/ (accessed October 12, 2007).

Kripke, S. (1959) A Completeness theorem in modal logic. *The Journal of Symbolic Logic* 24: 1, 1–14.

Leibniz, G. W. (1964) *The Existence of God*. Ed. J. Hick. New York: Macmillan. Reprinted from *New Essays Concerning Human Understanding* by G. W. Leibniz (1896). Trans. A. G Langley. New York: Macmillan, Appendix X.

Malcolm, N. (1967) Anselm's ontological arguments. In J. H. Hick and A. C. McGill (eds.), *The Many-Faced Argument*, 301–20. New York: Macmillan. Reprinted from *The Philosophical Review* LXIX: 1 (1960).

Maydole, R. (1980) A modal model for proving the existence of God. *American Philosophical Quarterly* 17, 135–42.

Maydole, R. (2000) "The Modal Third Way." *International Journal for Philosophy of Religion* 47: 1–28.

Maydole, R. (2003) The modal perfection argument for the existence of God. *Philo* 6, 299–313.

Maydole, R. (2005a) On Oppy's objections to the modal perfection argument. *Philo* 8: 2, 123–130.

Maydole, R. (2005b) On Metcalf's objections to the modal perfection argument. *Philo* 8: 2, 134–6.

McGill, A. C (1967) Recent discussions of Anselm's argument. In J. H. Hick and A. C. McGill (eds.), *The Many-Faced Argument*, 33–110. New York: Macmillan.

Metcalf, T. (2005) Entailment and ontological arguments: reply to Maydole, *Philo* 8: 2, 131–3.

Oppy, G. (1995) *Ontological Arguments and Belief in God*. Cambridge: Cambridge University Press.

Oppy, G. (1996) Gödelian ontological arguments. *Analysis* 56, 226–30.

Oppy, G. (2000) Response to Gettings. *Analysis* 60: 4, 363–7.

Oppy, G. (2004) Maydole's 2QS5 argument. *Philo* 77: 2, 203–11.

Oppy, G. (2007) Ontological Arguments. *Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/ontological-arguments/ (accessed July 29, 2007).

Plantinga, A. (1974) *The Nature of Necessity*. Oxford: Oxford University Press.

Rowe W. L. (2001) *Philosophy of Religion*, 3rd edn. Belmont, CA: Wadsworth.

Sobel, J. H. (1987) Gödel's ontological proof. In J. J. Thomson (ed.), *On Being and Saying: Essays for Richard Cartwright*, 241–61. Cambridge, MA: MIT Press.

Sobel, J. H. (2004) *Logic and Theism*. Cambridge: Cambridge University Press.

Appendix 1. Logic Matters

The strongest logic used in this chapter is a standard natural deduction system of second-order quantificational modal logic with identity (2QS5). It is equivalent to a standard second-order extension of Kripke's 1959 system of first-order modal logic. A weaker subsystem of 2QS5 is frequently used. The language of 2QS5 includes first- (lowercase) and second-order (uppercase) variables, constants and pseudo-names, property abstracts, and the standard array of quantifiers, connectives, punctuation marks, and so on.

The nonmodal propositional and quantificational inference rules of 2QS5 are from Gustason and Ulrich: Conjunction (Conj), Addition (Add), Simplification (Simp), Disjunctive Syllogism (DS), Excluded Middle Introduction (E-M I), Modus Ponens (MP), Modus Tollens (MT), Hypothetical Syllogism (HS), Constructive Dilemma (CD), Commutation (Com), Distribution (Dist), Association (Assoc) Double Negation (DN), DeMorgan (DeM), Transposition (Trans), Exportation (Exp), Equivalence (Equiv), Existential Instantiation (EI), Existential Generalization (EG), Universal Instantiation (UI), Universal Generalization (UG), Quantifier Negation (QN), Identity Introduction (II), Identity Elimination (IE), Conditional Proof (CP) and Indirect Proof (IP).[39]

The five Modal Inference Rules of 2QS5 are as follows:

	For every substitution instance of $\ulcorner p \urcorner$ and $\ulcorner q \urcorner$
NE (necessity elimination)	$\Box p$ /∴ p
MMP (modal modus ponens)	$\Box(p \supset q)$, $\Box p$ /∴ $\Box q$
NI (necessity introduction)	If $\ulcorner p \urcorner$ is a theorem then $\ulcorner \Box p \urcorner$ is a theorem
ME (modal equivalence)	$\ulcorner \Diamond p \urcorner$ for $\ulcorner \sim\Box\sim p \urcorner$ and $\ulcorner \Box p \urcorner$ for $\ulcorner \sim\Diamond\sim p \urcorner$
PN (possibility necessity)	$\Diamond p$ /∴ $\Box\Diamond p$

2QS5 also includes the Principle of Abstraction (Abs) as an axiom schema: $(x)(\hat{y}[\Psi y]x \equiv \Psi x)$, where $\ulcorner \hat{y}[\Psi y] \urcorner$ denotes the property of being a Ψ.[40]

Appendix 2. Formal Proofs of Some Modal Arguments

The validity of Malcolm's ontological argument

Let

$U_1x =_{df}$ x is unlimited
$D_1x =_{df}$ x depends on something else for its existence or nonexistence
$Hx =_{df}$ x happens to exist

39. The quantification rules apply to both first-order and second-order variables, and no pseudo-names are allowed in the last line of a deduction.
40. Theorems and theorem schemata of Q2S5 are sometimes used in the proofs and deductions in this chapter. It would make this chapter too long to include proofs of them. They are, however, fairly straightforward, and should be fairly common in the literature.

$Jx =_{df}$ x happens not to exist
$B_1x =_{df}$ x is caused to begin to exist by some other being
$C_1x =_{df}$ x is caused to cease to exist by some other being
$M_1x =_{df}$ x begins to exist
$N_1x =_{df}$ x ceases to exist

Deduction

1	$\Diamond Eg$	pr[41]
2	U_1g	pr
3	$(x)(U_1x \equiv (\sim D_1x \ \& \sim Hx \ \& \sim Jx))$	pr
4	$(x)(\sim D_1x \equiv (\sim B_1x \ \& \sim C_1x))$	pr
5	$(x)(M_1x \supset (B_1x \vee Hx))$	pr
6	$(x)(N_1x \supset (C_1x \vee Jx))$	pr
7	$(x)((\sim M_1x \ \& \sim N_1x) \supset ((Ex \supset \Box Ex) \ \& \ (\sim Ex \supset \Box \sim Ex)))$	pr
8	$U_1g \equiv (\sim D_1g \ \& \sim Hg \ \& \sim Jg)$	3, UI
9	$(\sim D_1g \ \& \sim Hg \ \& \sim Jg)$	2, 8 Equiv, Simp, MP
10	$\sim D_1g \equiv (\sim B_1g \ \& \sim C_1g)$	4 UI
11	$\sim D_1g$	9 Simp
12	$(\sim B_1g \ \& \sim C_1g)$	10, 11 Equiv, Simp, MP
13	$\sim Hg$	9 Assoc, Simp
14	$\sim B_1g$	12 Simp
15	$\sim (B_1g \vee Hg)$	13, 14 Conj, DeM
16	$\sim Jg$	9 Assoc, Simp
17	$\sim C_1g$	12 Simp
18	$\sim (C_1g \vee Jg)$	16, 17 Conj, DeM
19	$M_1g \supset (B_1g \vee Hg)$	5 UI
20	$\sim M_1g$	15, 19 MT
21	$N_1g \supset (C_1g \vee Jg)$	6 UI
22	$\sim N_1g$	18, 21 MT
23	$(\sim M_1g \ \& \sim N_1g) \supset ((Eg \supset \Box Eg) \ \& \ (\sim Eg \supset \Box \sim Eg))$	7 UI
24	$(\sim M_1g \ \& \sim N_1g)$	20, 22 Conj
25	$((Eg \supset \Box Eg) \ \& \ (\sim Eg \supset \Box \sim Eg))$	23, 24 MP
26	$(\sim Eg \supset \Box \sim Eg)$	25 Simp
27	$\sim \Box \sim Eg \supset Eg$	26 Trans
28	$\Diamond Eg \supset Eg$	27 ME
29	Eg	1, 28 MP

The validity of Hartshorne's ontological argument

Let

$q =_{df}$ There is a perfect being

Deduction

41. "Eg" could be replaced by "$(\exists x)x = g$." The existential quantifier would then have to have existential import, as it does for both Hartshorne and Plantinga.

1	$\Box(q \supset \Box q)$	pr
2	$\Diamond q$	pr
3	$\Box(q \supset \Box q) \supset (\Diamond q \supset \Diamond\Box q)$	theorem
4	$(\Diamond q \supset \Diamond\Box q)$	1, 3 MP
5	$\Diamond\Box q$	2, 4 MP
6	$\Diamond\Box q \supset \Box q$	theorem
7	$\Box q$	5,6 MP
8	q	7, NE

The validity of Plantinga's ontological argument

Let

$Ax =_{df}$ x is maximally great
$Bx =_{df}$ x is maximally excellent
$W(Y) =_{df}$ Y is a universal property
$Ox =_{df}$ x is omniscient, omnipotent, and morally perfect

Deduction

1	$\Diamond(\exists x)Ax$	pr
2	$\Box(x)(Ax \equiv \Box Bx)$	pr
3	$\Box(x)(Bx \supset Ox)$	pr
4	$(Y)[W(Y) \equiv (\Box(\exists x)Yx \vee (\Box\sim(\exists x)Yx)]$	pr
5	$(Y)[(\exists Z)\Box(x)(Yx \equiv \Box Zx) \supset W(Y)]$	pr
6	$(\exists Z)\Box(x)(Ax \equiv \Box Zx)$	2, EG
7	$[(\exists Z)\Box(x)(Ax \equiv \Box Zx) \supset W(A)]$	5, UI
8	$W(A) \equiv (\Box(\exists x)Ax \vee (\Box\sim(\exists x)Ax)$	4, UI
9	$W(A)$	6, 7 MP
10	$W(A) \supset (\Box(\exists x)Ax \vee (\Box\sim(\exists x)Ax)$	8, Equiv, Simp
11	$\Box(\exists x)Ax\ (\Box\sim(\exists x)Ax)$	9, 10 MP
12	$\sim\Diamond\sim\sim(\exists x)Ax \vee (\Box(\exists x)Ax)$	11, Com, ME
13	$\Diamond(\exists x)Ax \supset \Box(\exists x)Ax$	DN, Impl
14	$\Box(\exists x)Ax$	1, 13 MP
15	$\Box(x)(Ax \equiv \Box Bx) \supset (\Box(\exists x)Ax \supset \Box(\exists x)\Box Bx)$	theorem
16	$\Box(\exists x)\Box Bx$	14, 15 MP (twice)
17	$\Box(x)(Bx \supset Ox) \supset (\Box(\exists x)\Box Bx \supset \Box(\exists x)\Box Ox)$	theorem
18	$\Box(\exists x)\Box Ox$	16, 17 MP (twice)
19	$(\exists x)\Box Ox$	18, NE

Complete proofs of Gödel's ontological theorems

Proof of Tm 1

1	$\sim\Diamond(\exists y)G_1y$	AIP
2	$\sim\Diamond(\exists y)G_1y \supset \Box(x)(G_1x \supset \sim G_1x)$	theorem

3	$\Box(x)(G_1x \supset {\sim}G_1x)$	1, 2 MP
4	$\Box(x)(\hat{y}[{\sim}G_1y]x \equiv {\sim}G_1x)$	Abs, NI[42]
5	$(\Box(x)(G_1x \supset {\sim}G_1x)\ \&\ \Box(x)(\hat{y}[{\sim}G_1y]x \equiv {\sim}G_1x)) \supset$ $\Box(x)(G_1x \supset \hat{y}[{\sim}G_1y]x)$	theorem
6	$\Box(x)(G_1x \supset \hat{y}[{\sim}G_1y]x)$	3, 4, 5, Conj, MP
7	$P_1(G_1)$	Ax 3[43]
8	$(P_1(G_1)\ \&\ \Box(x)(G_1x \supset \hat{y}[{\sim}G_1y]x)) \supset P_1\hat{y}[{\sim}G_1y])$	Ax 2, UI
9	$P_1\hat{y}[{\sim}G_1y]$	6, 7, 8, Conj, MP
10	$P_1(G_1) \supset {\sim}P_1(\hat{y}[{\sim}G_1y])$	Ax 1, UI, Equiv, Simp
11	${\sim}P_1(\hat{y}[{\sim}G_1y])$	9, 10 MP
12	$\Diamond(\exists y)G_1y$	1–11 IP

Proof of Tm 2

1	$G_1\mu\ \&\ \Theta\mu$	ACP
2	${\sim}P_1(\Theta)$	AIP
3	${\sim}P_1(\hat{y}[{\sim}\Theta y]) \supset P_1(\Theta)$	Ax 1, UI, Equiv, Simp
4	$P_1(\hat{y}[{\sim}\Theta y])$	2, 3, DN, MT
5	$P_1(\hat{y}[{\sim}\Theta y]) \supset \hat{y}[{\sim}\Theta y]\mu$	1, Simp, df "G_1", UI
6	$\hat{y}[{\sim}\Theta y]\mu$	4, 5 MP
7	$\hat{y}[{\sim}\Theta y]\mu \equiv {\sim}\Theta\mu$	Abs, UI
8	${\sim}\Theta\mu$	6, 7 Equiv, Simp, MP
9	$\Theta\mu\ \&\ {\sim}\Theta\mu$	1, 8 Simp, Conj
10	$P_1(\Theta)$	2–7 IP
11	$\Box(x)(G_1x \supset (Y)(P_1(Y) \supset Yx))$	theorem, df "G_1"
12	$\Box(x)(G_1x \supset (Y)(P_1(Y) \supset Yx)) \supset (x)(Y)(\Box P_1(Y) \supset$ $\Box(x)(G_1x \supset Yx))$	theorem
13	$(x)(Y)(\Box P_1(Y) \supset \Box(x)(G_1x \supset Yx))$	11, 12 MP
14	$\Box P_1(\Theta) \supset \Box(x)(G_1x \supset \Theta x)$	13 UI
15	$P_1(\Theta) \supset \Box P_1(\Theta)$	Ax 4, UI
16	$\Box P_1(\Theta)$	10, 15 MP
17	$\Box(x)(G_1x \supset \Theta x)$	14, 16 MP
18	$(G_1\mu\ \&\ \Theta\mu) \supset \Box(x)(G_1x \supset \Theta x))$	1–17 CP
19	$(x)(Z)((G_1x\ \&\ Zx) \supset \Box(x)(G_1x \supset Zx))$	18, UG
20	$(x)(Z)((G_1x\ \&\ Zx) \supset \Box(x)(G_1x \supset Zx)) \supset$ $(x)(G_1x \supset (G_1x\ \&\ (Z)(Zx \supset \Box(x)(G_1x \supset Zx))))$	theorem
21	$(x)(G_1x \supset (G_1x\ \&\ (Z)(Zx \supset \Box(x)(G_1x \supset Zx))))$	19, 20, MP
22	$(x)(G_1x \supset E_1(G_1, x))$	21, df "E_1"

42. Scott and Sobel omit necessitated Abstraction in their proofs of Tm 1. (Anderson acknowledges that it is implicit.) Their proofs assume that it is necessarily true that self-difference is identical to the negation of self-identity. But howsoever obvious that identity might be, its proof requires necessitated Abstraction:

$$\Box(x)(\hat{y}[y{\neq}y]x \equiv {\sim}x{=}x)$$
$$\Box(x)(\hat{y}[{\sim}y{=}y]x \equiv {\sim}x{=}x)$$
$$\therefore\ \hat{y}[y{\neq}y] = \hat{y}[{\sim}y{=}y]$$

43. See section (4a) for a formal expression of Gödel's axioms.

Proof of Tm 3

1	$G_1\mu$	ACP
2	$P_1(N_2)$	Ax 5
3	$P_1(N_2) \supset N_2\mu$	1, UI, df "G_1"
4	$N_2\mu$	2, 3 MP
5	$E_1(G_1, \mu) \supset \Box(\exists z)G_1z$	4, UI, df "N_2"
6	$G_1\mu \supset (G_1\mu \;\&\; (Z)(Zx \supset \Box(x)(G_1\mu \supset Z\mu)))$	Tm 2, df "E_1", UI
7	$(G_1\mu \supset (G_1\mu \;\&\; (Z)(Zx \supset \Box(x)(G_1\mu \supset Z\mu)))) \supset$ $(G_1\mu \;\&\; (Z)(Zx \supset \Box(x)(G_1\mu \supset Z\mu)))$	theorem
8	$E_1(G_1, \mu)$	6, 7 MP, df "E_1"
9	$\Box(\exists z)G_1z$	5, 8 MP
10	$G_1\mu \supset \Box(\exists z)G_1z$	1–9 CP
11	$\Box(x)(G_1x \supset \Box(\exists z)G_1z)$	10 UG, NI
12	$\Box(x)(G_1x \supset \Box(\exists z)G_1z) \supset (\Diamond(\exists y)G_1y \supset \Diamond\Box(\exists z)G_1z)$	theorem
13	$\Diamond(\exists y)G_1y \supset \Diamond\Box(\exists z)G_1z$	11, 12 MP
14	$\Diamond\Box(\exists z)G_1z$	13 Tm 1, MP
15	$\Diamond\Box(\exists z)G_1z \supset \Box(\exists z)G_1z$	theorem
16	$\Box(\exists z)G_1z$	14, 15 MP
17	$(\exists z)G_1z$	16 NE