A Gödelian Ontological Argument Improved

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December 4, 2007

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I shall defend several versions of Gödel's ontological argument that use weaker premises than the version of Anderson (1990), and that, like Anderson's argument, avoid modal collapse. Moreover, the versions of the argument that I shall defend are not subject to Oppy's (1996 and 2000) parody refutations.

The basic primitive notion in a Gödelian argument is that of a *positive property*. We can understand that in several different ways, each one giving rise to a different interpretation of the argument. For instance, one might take a positive property to be one that in no respect detracts from any respect of the excellence (or greatness or value, depending on how we prefer to phrase it) of the entity that has the property but whose negation does detract from some respect of the excellence (or greatness or value) of the possessor. Or one might take a positive property to be one that does not entail any limitation but whose negation does. Or one might start with a somewhat Leibnizian structure of the space of properties, on which there are some *basic* properties that are mutually compatible (e.g., because they are logically independent of each other), and then count a property as positive provided that it is entailed by at least one of the basic properties.

Each of these interpretations makes plausible the following two "formal" axioms:

Axiom F1. If *A* is positive, then ~*A* is not positive.

Axiom F2. If *A* is positive and *A* entails *B*, then *B* is positive.

The correctness of F1 on the excellence, goodness, greatness and no-limitation readings is clear, and on the Leibnizian interpretation F1 follows from the compatibility of the basic properties. Moreover, if a property doesn't detract from the excellence (or goodness or greatness) of an entity, then anything it entails had better not detract from it either. On the other hand, if a property detracts from the excellence (or goodness or greatness) of an entity, so does any property that entails that property. Hence if ~*A* detracts from excellence (etc.), and *A* entails *B*, then ~*B* detracts from excellence (etc.), since ~*B* entails ~*A* by contraposition. This yields Axiom F2 on the excellence, goodness and greatness interpretations. Exactly the same reasoning shows that if a property does not entail any limitation but its negation does, the same holds for any property that it entails. And closure under entailment is trivial on the Leibnizian interpretation, so F2 follows once again.

All of the Gödelian arguments I shall offer will make use of the "formal" axioms F1 and F2. In addition, all the arguments will make use of the following "non-formal" axiom:

Axiom N1. Necessary existence is positive,

where necessary existence (N.E.) is defined as follows:

Definition D1. x has N.E. iff $*F*[ÿ(*x* has *F*) and ÿ$*y*(*y* has *F*) and ÿ"*y*(*y* has *F* É *y*=*x*)].

Note that D1 makes necessary existence logically weaker than on Gödel's original definition, since Gödel's definition entailed that every property of a necessarily existing being was an essential property of it (Koons, 2005). Gödel's analogue of N1 together with Axiom F2 will entail ourAxiom N1, so our axioms are still weaker than Gödel's. I will call a being that has necessary existence a "necessary being".

We need one more definition for our first ontological argument:

Definition D2. A property *A* is *strongly positive* iff ÿ(*having A essentially* is a positive property).

I shall abbreviate the property of *having A essentially* as *EA*. Then D2 says that *A* is strongly positive iff necessarily *EA* is positive. Given Axiom F1, a strongly positive proposition is positive.

I shall assume a modal logic that includes S5. As is usual in modal logic, axioms, and hence theorems as well, are always taken to be necessary truths. Moreover, I shall assume that all properties exist in all worlds, i.e., that properties exist necessarily. Because of this, it is possible to interchange the order of modal operators and quantifications over properties.

At this point, we are ready to state the simplest ontological argument:

Theorem T1. Given Axioms F1, F2 and N1, if *A* is any strongly positive property, then there exists a being that exists necessarily and essentially has *A*.

It is rather surprising that T1 follows from F1, F2 and N1 without any further assumptions. Gödelian arguments typically assume additional axioms, e.g., the axiom that to have all and only positive properties as essential properties is a positive property (a claim stronger than F3, below). The proof of T1 is in the Appendix, but to assuage curiosity on the part of the reader, I will prove a crucial and simple lemma here:

Lemma L1. Given Axioms F1 and F2, if *A* and *B* are positive, then *A* and *B* are compossible, where:

Definition D3. A collection *C* of properties is compossible iff possibly there is an entity that exemplifies all the members of *C*.

The proof of L1 is very easy. If we assume for a *reductio* that *A* and *B* are incompossible, then *A* entails ~*B*. By F1, ~*B* is then positive, but this contradicts F2. Given L1, the proof of T1, whose details are in the Appendix, depends on applying L1 to the pair of positive properties *EA* and N.E. to conclude that possibly there is a necessary being that essentially has *A*, and then using S5 to eliminate the “possibly”.

How useful Theorem T1 is as a theistic argument depends on what kinds of strongly positive properties one thinks there are. The following non-formal axiom is plausible:

Axiom N2. Each of the properties of *essential omnipotence*, *essential omniscience* and *essential perfect goodness* is positive.

If it is an axiom that *EA* is positive, then necessarily *EA* is positive, and hence *A* is strongly positive. Thus, given N2, we immediately get the following result from T1:

Corollary C1. Given Axioms F1, F2, N1 and N2, there exists an essentially omnipotent necessary being, and there exists an essentially omniscient necessary being, and there exists an essentially perfectly good necessary being.

It would be nice if we could prove that one and the same necessary being is essentially omnipotent, omniscient and perfectly good. But I do not think this can be shown from F1, F2, N1 and N2 alone, though of course the claim that the three existential claims in C1 are each made true by the same being is *prima facie* compatible with these axioms. However, C1 is already quite interesting. Presumably no atheist accepts any of the conjuncts in the conclusion of C1. Moreover, if we can further argue that there are logical connections between divine attributes, we can get a little more. For instance, if a being is essentially omnipotent, then at least it has the essential property of *being able to* figure out the truth value of each proposition if it should want to, and so we might conclude that there necessarily exists a being that is essentially omnipotent and essentially able to figure out all facts that it wants to figure out.

To do better, we need a formal axiom about how positive properties can be conjoined. There are two options available. First, we might, not implausibly, suppose that the conjunction of all strongly positive properties is positive:

Axiom F3. The property of having all strongly positive properties is a positive property.

Given F1, this axiom is weaker than the corresponding axiom in the Anderson version of the Gödelian argument, which axiom stated that the property of having all *and only* the positive properties as essential properties is positive, since the latter property entails the property of having all strongly positive properties that figures in F3. Moreover, F3 is a bit more intuitive than Anderson's axiom. It is not obvious, for instance, that *lacking* any essential properties beyond the positive ones is positive. Moreover, the full Anderson axiom is essential to the parodies in Oppy (1996, 2000), while the present argument does not appear to be subject to those parodies (though I have no argument that other parodies cannot be constructed).

We now have the following result (see the Appendix for the proof):

Theorem T2. Given Axioms F1-F3 and N1, there is a necessary being that essentially has all strongly positive properties.

One might, however, object to F3 on the following grounds. Axiom F3 seems innocent until we realize the following fact, which follows immediately from Lemma L1 in the special case where $A=B$:

Lemma L2. Given Axioms F1 and F2, any positive property A is possibly exemplified.

Thus, F1, F2 and F3 entail that the conjunction of all strongly positive properties is possibly exemplified, and hence that all strongly positive properties are compossible. But the latter is quite a strong claim, and one might worry whether an intuition that so quickly entails it can be that plausible. One might also worry that F3 is more than just a formal claim about the logic of positive properties, since it seems to make the substantive claim that a particular property is positive, and hence is more akin to the non-formal axioms N1 and N2, and such non-formal axioms are likely more vulnerable.

For these reasons, it will be better to make use of the following "more formal" axiom:

Axiom F4. If *A* and *B* are strongly positive and compossible, then their conjunction is positive.

As far as intuitive support goes, we could probably have gone with "positive" in place of "strongly positive" in the antecedent, but since putting "strongly positive" in the antecedent will yield a weaker set of axioms (since by F1 any strongly positive property is positive), and F4 is all I will need, I might as well go with F4. Observe that F3 together with F1 entails F4, so using F1, F2 and F4 will be using a weaker set of assumptions than just using F1, F2 and F3.

Now we have a result that, while not quite as satisfying as T2, is in practice about as useful to a theist:

Theorem T3. Given Axioms F1, F2, F4 and N1, if *U* is any finite set of strongly positive properties, then there is a necessary being that essentially has every member of *U*.

We can conclude from this:

Corollary C2. Given Axioms F1, F2, F4, N1 and N2, there is a necessary being that is essentially omniscient, essentially omnipotent and essentially perfectly good.

The same holds with F3 in place of F4, either because we then have a weaker set of axioms, or by using T2 instead of T3.

It would be nice if one could drop the assumption of finiteness in T3, but I doubt this can be done without adding an axiom like F4. The conclusion of T3 is at least *prima facie* compatible with the stronger claim that there is a necessary being that essentially has all strongly positive properties. Even though T3 may not give the theist all that theist wants, there may be no harm in leaving to faith the claim that God has *all* the strongly positive properties. And C2 is quite respectable, surely, and more than sufficient to refute atheism.

The ontological arguments embodied in T1, T2, T3, C1 and C2 are valid. Each of Theorems T1, T2 and T3 uses a weaker (at least *prima facie*) set of axioms than the best previous Gödelian argument, that of Armstrong. In particular, the problem of modal collapse is avoided in these theorems just as it is in Armstrong's work.

Whether the arguments are sound will, I think, depend on a deeper analysis of the nature of positivity. But the premises all appear at least somewhat *plausible*, are largely independent of the premises of non-ontological arguments such as the cosmological argument or the argument from religious experience, and hence the Gödelian arguments should further lower the probability of atheism and increase that of theism.

Appendix: Proofs of the Theorems

First we prove T1. To that end, we need two auxiliary results:

Lemma L3. If *a* has N.E., then ÿ(*a* has N.E.).

Proof of L3: By S4, if ÿ*p*, then ÿÿ*p*. The truth of L3 thus easily follows from S4 and D1. ¨

Lemma L4. For any property *B*, if $*x*(*x* has N.E. and *x* has *B*), then ÿ$*x*(*x* has N.E. and à(*x* has *B*)).

Proof of L4: Suppose the antecedent of the claim in L4. By existential instantiation, let *a* be such that *a* has N.E. and *a* has *B*. Then, *a* has N.E. necessarily by L3. Moreover, by the Brouwer axiom of modal logic, if *a* has *B*, then ÿà(*a* has *B*). Hence, ÿ(*a* has N.E. and à(*a* has *B*)). It follows from this that $*x*ÿ(*x* has N.E. and à(*x* has *B*)). But if $*x*ÿ*Fx*, then ÿ$*xFx*, and the proof is complete. ¨

Proof of T1: Let *P* be the conjunction of N.E. and *EA*. By Lemma L4, possibly there is an *x* that satisfies *P*. Thus, possibly, there is an *x* that has N.E. and *EA*. Now L4 is a necessary truth, since it is a theorem of modal logic, so it follows from the necessity of L4 that if à$*x*(*x* has N.E. and *x* has *B*), then àÿ$*x*(*x* has N.E. and à(*x* has *B*)). Using S5 and substituting *EA* for *B*, we conclude that there is an *x* that has N.E. and that possibly essentially has *A*. But by S5, if one possibly essentially has *A*, then one essentially has *A* (accessibility is an equivalence relation given S5, so if *x* possibly essentially has *A*, then *x* essentially has *A* in some accessible world *w*, and hence *x* has *A* in every world accessible from *w* at which *x* exists, which comes to the same thing as saying that *x* has *A* in every accessible world at which *x* exists). ¨

Theorems T2 and T3 both follow from T1 in a fairly easy way. First, we need the following result:

Lemma L5. Given F1, if *A* is strongly positive, so is *EA*.

Proof of L5: *EA* entails *EEA*, given axiom S4. If *A* is strongly positive, *EA* is necessarily positive, and by F1 (which holds necessarily), so is *EEA*. Hence *EA* is strongly positive. ¨

Now we can prove T2.

Proof of T2: Let *P* be the property of having all properties of the form *EA* where *A* is a strongly positive property. By L5, having every strongly positive property entails having *EA* for every strongly positive property *A*. Thus, *P* is entailed by the conjunctive property in F3, and hence *P* is positive by F1 and F3. By T1, it follows that there is a necessary being that essentially has *P*, and T2 follows. ¨

To prove T3, we first need the following result:

Lemma L6. Given F1, F2 and F4, the conjunction of any finite set of strongly positive properties is strongly positive.

Proof of L6: It suffices to show that if *A* and *B* are strongly positive, then the conjunction *A*&*B* is strongly positive as well, since we can then repeat that argument to get the strong positivity of the conjunction of all the members of the finite set. So suppose *A* and *B* are strongly positive. Then *EA* and *EB* are strongly positive by L5, and hence compossible by L1. By F4, we conclude that *EA*&*EB* is positive, and since F4 holds necessarily, we in fact conclude that *EA*&*EB* is necessarily positive. But *EA*&*EB* entails *E*(*A*&*B*), and so by F1, we conclude that *E*(*A*&*B*) is necessarily positive, and thus *A*&*B* is strongly positive. ¨

Proof of T3: Let *P* be the conjunction of the properties in *U*. Then, *P* is strongly positive by L6, and the conclusion of T3 follows from T1. ¨

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