ANIMAL cognition and desire, from the appetite of a clam to the optical systems of vultures and frigate birds, is supposed to have neurobiological explanations resultant from, if not reducible to, universal laws of physics. That is a minimal and modest project for epistemology naturalized, one to be assisted by specialized sciences.

There is a larger and bolder project of epistemology naturalized, namely, to explain human thought in terms available to physical science, particularly the aspects of thought that carry truth values, and have formal features, like validity or mathematical form. That project seems to have hit a stone wall, a difficulty so grave that philosophers dismiss the underlying argument, or adopt a cavalier certainty that our judgments only simulate certain pure forms and never are real cases of, e.g., conjunction, *modus ponens*, adding, or genuine validity. The difficulty is that, in principle, such truth-carrying thoughts cannot be wholly physical (though they might have a physical medium), because they have features that no physical thing or process can have at all.

1 After three centuries of amazingly successful science, we do not have a successful explanation of animal cognition, not even for a spider or a fish. Probably, we have been misconceiving the project in ways that makes science both less productive and less helpful.

2 Thinking here means "judgmental understanding"—what Aristotle thought to be the actuality of the intellect (*De Anima*, bk. III, ch. 4, 429b, 30: "Mind is in a sense potentially whatever is thinkable, though actually it is nothing until it has thought"). There are many kinds of thinking; some thinkings are bodily doings, like my pouring a liquid. But it is only the processes of understanding that I am now trying to show cannot be wholly physical; understandings that involve feeling cannot be entirely nonphysical either, any more than my going for a walk can be a mere willing.

3 See Aristotle’s argument (*De Anima*, bk. III, ch. 4, 429a, 10–28; see also Aquinas’s commentary in *Aristotle’s De Anima in the Version of William of Moerbeke and the Commentary of St. Thomas Aquinas*, Kenelm Foster and Silvester Humphries, trans. (New Haven: Yale, 1959 repr.), sec. 684–6, pp. 406–7) that the understanding cannot have an organ as sight has the eye (and nowadays philosophers suppose thinking has the brain), because the limited physical states of an organ would fall short of the contrasting states of understanding that we know we can attain.

4 Philosophers should not recoil with distaste at such remarks about thought, because they attribute even odder features to propositions, e.g., being infinite in number, belonging to a tight logical network with formal features like "excluded middle," and being such that every one is determinately either logically related, by implication or exclusion, or logically independent of every other; in fact, in a system of material implication, no proposition is logically independent of any other.
I propose to articulate that “difficulty in principle” so as to press home the point that it cannot be dismissed or evaded, or the underlying arguments or costs disregarded. First, the underlying arguments themselves are among the jewels of analytic philosophy (underdetermination considerations); and, secondly, to deny that our judgments are of definite logical forms and pure functions conflicts with our own certainty and with what we tell our logic, mathematics, and linguistics students about validity, proof, and formal syntax, and leaves us unable to explain what we do when we do mathematics, logic, or any other formal thinking.

But now let us look at the argument:

Some thinking (judgment) is determinate in a way no physical process can be. Consequently, such thinking cannot be (wholly) a physical process. If all thinking, all judgment, is determinate in that way, no physical process can be (the whole of) any judgment at all. Furthermore, “functions” among physical states cannot be determinate enough to be such judgments, either. Hence some judgments can be neither wholly physical processes nor wholly functions among physical processes.

Certain thinking, in a single case, is of a definite abstract form (e.g., $N \times N = N^2$), and not indeterminate among incompossible forms (see I below). No physical process can be that definite in its form in a single case. Adding cases even to infinity, unless they are all the possible cases, will not exclude incompossible forms. But supplying all possible cases of any pure function is impossible. So, no physical process can exclude incompossible functions from being equally well (or badly) satisfied (see II below). Thus, no physical process can be a case of such thinking. The same holds for functions among physical states (see IV below).

I. THE DETERMINATENESS OF SOME THOUGHT PROCESSES

Can judgments really be of such definite “pure” forms? They have to be; otherwise, they will fail to have the features we attribute to them and upon which the truth of certain judgments about validity, inconsistency, and truth depend; for instance, they have to exclude incompossible forms or they would lack the very features we take to be definitive of their sorts: e.g., conjunction, disjunction, syllogistic, modus ponens, etc. The single case of thinking has to be of an abstract “form” (a “pure” function) that is not indeterminate among incompossible ones. For instance, if I square a number—not

5 But in part, yes, in the sense that my utterances are physical. Moreover, the thought may not even be possible apart from feeling or sense, just as a gesture is not possible without bodily movement. The target in this paper is theories that thoughts are “no more than” physical or functions determined physically; not that, for us, they are “at least physically realized.”
just happen in the course of adding to write down a sum that is the square, but if I actually square the number—I think in the form “\( N \times N = N^2 \).”

The same point again. I can reason in the form, modus ponens (“If \( p \) then \( q \)”; “\( p \)”; “therefore, \( q \)”). Reasoning by modus ponens requires that no incompossible form also be “realized” (in the same sense) by what I have done. Reasoning in that form is thinking in a way that is truth-preserving for all cases that realize the form. What is done cannot, therefore, be indeterminate among structures, some of which are not truth preserving.\(^6\) That is why valid reasoning cannot be only an approximation of the form, but must be of the form. Otherwise, it will as much fail to be truth-preserving for all relevant cases as it succeeds; and thus the whole point of validity will be lost. Thus, we already know that the evasion, “We do not really conjoin, add, or do modus ponens but only simulate them,” cannot be correct. Still, I shall consider it fully below.

“Being truth preserving for all relevant cases” is a feature of the single case. The form of the reasoning that actually occurs is “truth-preserving,” regardless of which case it is. Otherwise, it would not be “impossible by virtue of the form to proceed from truth to falsity” in that reasoning (especially when the premises are not true). Thus, the form of the actual “encompasses” (logically contains) all relevant counterfactual situations. In fact, it encompasses all relevant cases whatever. Without that, there is no genuine difference between valid and invalid reasoning.

Squaring, conjoining, adding. I propose with some simple cases to reinforce the, perhaps already obvious, point that the pure function has to be wholly realized in the single case, and cannot consist in the array of “inputs and outputs” for a certain kind of thinking. Does anyone doubt that we can actually square numbers? “4 times 4 is sixteen”; a definite form \( N \times N = N^2 \) is “squaring” for all relevant cases, whether or not we are able to process the digits, or talk long enough to give the answer. To be squaring, I have to be doing something which works for all the cases, something for which any relevant case can be substituted without change in what I am doing, but only in which thing is done.

Size and length of computation, for example, are external to the form of thinking, accidental to what is done. I am squaring just in

\(^6\) I am not, of course, suggesting that a valid course of reasoning is not also a case of a variety of invalid forms, e.g., “\( P \), therefore, \( C \)” But it must determinately be a case of some valid form.
case my thinking is of the form mentioned. If it is of any incomposable form, or is indeterminate among incompossible forms, it is not of the form, “\(N\) times \(N = N\) squared.” It is not then squaring, however much its products may look like it, and however long a sequences of its outputs do.

The fact that I cannot process every case of modus ponens, because most of them have premises too long for me to remember, sentences too long to say, or words I do not understand, is adventitious, like my not being able to do modus ponens in Portuguese. Those are features of the functors, not of the function. The function that has to be realized in every case is the one wholly realized in the single case.

That point is to be taken literally: that the function is wholly present, not by approximation, exemplification, or simulation, but by realization in the single case. To make that distinction clearer, consider an even simpler function, “conjoining.” Conjoining is the functional arrangement of an \(n\)-tuple of assertions into a single assertion that is determinately true just in case every one of the \(n\)-tuple of judgments is, and false otherwise. The truth of the whole block is the truth of all of the units (“\(p \cdot q = T\) just in case \(p = T\) and \(q = T\)”). I can conjoin every sentence in the fourteenth edition of the Encyclopedia Britannica, or yesterday’s Times. What I do in the single case is what would conjoin any string of suitable units, even ones too long for me to think of, or beyond my access to refer to. It is impossible to conjoin thoughts, if what I do is indeterminate among incompossible forms (at the same level).

Adding—genuinely adding, not estimating—is a sum-giving thought form for any suitable array of numbers.\(^7\) If I add two “elevens,” I am doing what would have given “forty-four” had I been adding two “twenty-twos” (and not making mistakes), and so on for every other combination of suitable numbers. I cannot be really adding when I do something which gives the “right output” but which cannot, by its form, determine the “right outcome” for any case whatever, even one on which I make a mistake. There is a great difference between adding incorrectly and doing something else, like guessing, estimating, or following a routine or algorithm.

\(^7\) Some conjunction tasks seem possible that are not: e.g., to conjoin all statements that can be expressed in English. That impossibility is not because of some fuzziness about the function “conjoin,” but because the supposed totality is incoherent. You cannot add up all the even numbers, taken pairwise, just as you cannot conjoin all the sentences of English. See note 10.
The adding I am talking about, like conjoining, is a form of understanding. This is not a claim about how many states we can be in. This is a claim about the ability exercised in a single case, the ability to think in a form that is sum-giving for every sum, a definite thought form distinct from every other. When a person has acquired such an ability is not always transparent from successful answers, and it can be exhibited even by mistakes.

Definite forms of thought are dispositive for every relevant case actual, potential, and counterfactual. Yet the “function” does not consist in the array of inputs and outcomes. The function is the form by which inputs yield outputs. The array of inputs and outputs for a function is the logical tail of the comet, not what the function is.

The trait that determines the tail of the comet, the trait that “settles every relevant case, including all countercases,” marks the contrast with any physical process: a physical process has no feature that can do that. That grounds my main argument: that a necessary consequence of even a single case of such thinking is something that is logically impossible to be a consequence of any physical process, or function among physical processes, whatever. Thus, the activity of such thinking cannot be a physical process, and the ability for such thinking cannot be a physical capacity.

II. THE INDETERMINACY OF THE PHYSICAL

Now we need reasons why no physical process or function among physical processes can determine “the outcome” for every relevant case of a “pure” function. Those considerations mark some of the most successful analytic philosophy, from W. V. Quine, to Nelson Goodman, to Saul Kripke. No physical process is so definite as to determine among incompossible abstract functions that one rather than another is realized, and thus to settle for every relevant case what the “outcome” is to be. That indeterminacy remains no matter

---

8 We can even add certain nonterminating decimals, like .33333 and .66666 carrying from infinity to get 1. That is a form of understanding.

9 Equivalent but nonsynonymous functions would give the same arrays from inputs to outputs. Besides, a device that went to an address for the answer, and took it out in an envelope (encoded), which it did not open (decode) but handed to you (displayed for you to decode), could be made to produce the same array of outputs as addition. Yet it would not be adding. Besides, look at this function: 10 Z = X*X*X; 20 Print Z; 30 X = X + 1; 40 GOTO 10. That is a machine function for an endless loop to print the cube of every number beginning with zero. You can see that no matter what outputs the machine gives, it might have been doing something other than printing successive cubes, unless it produces all cubes—which cannot be done.
how long the physical process is "repeated," even infinitely. In a word, with a machine it is indeterminate among incompossible functions what it is doing, no matter what it does.\textsuperscript{10} Therefore, no matter what it does, what it is doing remains formally indeterminate. Goodman's\textsuperscript{11} "grue" considerations and the plus-quus adaptations by Kripke\textsuperscript{12} suggest the form of my argument to show that. The argument is as follows.

Whatever the discriminable features of a physical process may be, there will always be a pair of \textit{incompatible} predicates, each as empirically adequate as the other, to name a function the exhibited data or process "satisfies." That condition holds for any finite actual "outputs," no matter how many. That is a feature of physical process itself, of change. There is nothing about a physical process, or any repetitions of it, to block it from being a case of incompossible forms ("functions"), if it could be a case of any pure form at all. That is because the differentiating point, the point where the behavioral outputs diverge to manifest different functions, can lie beyond the actual, even if the actual should be infinite; e.g., it could lie in what the thing would have done, had things been otherwise in certain ways. For instance, if the function is $x(*y) = (x + y, \text{if } y < 10^{40} \text{ years}, = x + y + 1, \text{otherwise})$, the differentiating output would lie beyond the conjectured life of the universe.

Just as rectangular doors can approximate Euclidean rectangularity, so physical change can simulate pure functions but cannot realize them. For instance, there are no physical features by which an adding machine, whether it is an old mechanical "gear" machine or a hand calculator or a full computer, can exclude its satisfying a function incompatible with addition, say, quaddition (cf. Kripke's solution to the Riddle of Induction).

\textsuperscript{10} Postulating an infinity of cases will not suitably discriminate the functions that are the same for even numbers but differ for odd numbers after $N$. Postulating that "all" the cases are actual involves an \textit{incoherent totality}, because the machine cannot both do all that it does \textit{and} all that it might have done instead. Consequently, a pure function does not reduce to a pattern of inputs and outputs.

"All the additions" is as incoherent as "all the sets." So "what" addition is cannot be explained by "all the outcomes": rather, each and every outcome is determined by what addition is. It is impossible that all cases of addition be actual, even if infinities are performed because, even if we used up all the suitable numbers, the function itself would still be repeatable, say, for the same additions, but now done in a different order. The function cannot be exhausted by its cases, however many there are.


definition (op. cit., p. 9) of the function to show the indeterminacy of the single case: quus, symbolized by the plus sign in a circle, “is defined by: \( x \oplus y = x + y \), if \( x, y < 57 \), \( =5 \) otherwise”) modified so that the differentiating outputs (not what constitutes the difference, but what manifests it) lie beyond the lifetime of the machine. The consequence is that a physical process is really indeterminate among incompatible abstract functions.

Extending the list of outputs will not select among incompatible functions whose differentiating “point” lies beyond the lifetime (or performance time) of the machine. That, of course, is not the basis for the indeterminacy; it is just a grue-like illustration. Adding is not a sequence of outputs; it is summing; whereas if the process were quadding, all its outputs would be quadditions, whether or not they differed in quantity from additions (before a differentiating point shows up to make the outputs diverge from sums).

For any outputs to be sums, the machine has to add. But the indeterminacy among incompossible functions is to be found in each single case, and therefore in every case. Thus, the machine never adds.

Extending the outputs, even to infinity, is unavailing. If the machine is not really adding in the single case, no matter how many actual outputs seem “right,” say, for all even numbers taken pair-wise (see the qualifying comments in notes 7 and 10 about incoherent totalities), had all relevant cases been included, there would have been nonsums. Kripke drew a skeptical conclusion from such facts, that it is indeterminate which function the machine satisfies, and thus “there is no fact of the matter” as to whether it adds or not. He ought to conclude, instead, that it is not adding; that if it is indeterminate (physically and logically, not just epistemically) which function is realized among incompossible functions, none of them is. That follows from the logical requirement, for each such function, that any realization of it must be of it and not of an incompossible one.

There is no doubt, then, as to what the machine is doing. It adds, calculates, recalls, etc., by simulation. What it does gets the name of what we do, because it reliably gets the results we do (perhaps even more reliably than we do) when we add by a distinct process. The machine adds the way puppets walk. The names are analogous. The machine attains enough reliability, stability, and economy of output to achieve realism without reality. A flight simulator has enough realism for flight training; you are really trained, but you were not really flying.
A decisive reason why a physical process cannot be determinate among incompossible abstract functions is “amplified grueness”: a physical process, however short or long, however few or many outputs, is compatible with counterfactually opposed predicates; even the entire cosmos is. Since such predicates can name functions from “input to output” for every change, any physical process is indeterminate among opposed functions. This is like the projection of a curve from a finite sample of points: any choice has an incompatible competitor.

We have no doubt that the processes in a mechanical adding machine and in a personal computer are entirely physical. Addition cannot be identical with either of those physical processes because then it could not be done by the other. Suppose that addition is identical with a function among those processes. Then the processes would have to determine that function to the exclusion of every incompossible function. But they cannot do that, as the “quus,” “grue,” and “points-on-a-curve” examples show. So the machines cannot really add.

Secondly, opposed functions that are infinite (that is, are a “conversion” of an infinity of inputs into an infinity of outputs) can have finite sequences, as large as you like, of coincident outputs; they can even have subsequences that are infinitely long and not different (e.g., functions that operate “the same” on even numbers but differently on odd numbers). So for a machine process to be fully determinate, every output for a function would have to occur. For an infinite function, that is impossible. The machine cannot physically do everything it actually does and also do everything it might have done. That is the heart of the matter. The physical, as pro-

13 There is a complementary line of inquiry about immateriality. Christopher Cherniak argues (*Minimal Rationality* (Cambridge: MIT, 1986), p. 127) that because a physical object cannot be in an infinity of states, the mind treated as a brain computer is of limited understanding. That would be an understatement, were it true. Most of what actually happens would be unintelligible to us. An infinity of English sentences would be unintelligible, as would “most” truths of arithmetic.

For even if each of the finite number of electrochemical states the brain is capable of realizing actually happened, say, $10^{140}$ different thoughts, there would be an infinity of mathematical theorems we could not even understand because there would be no brain state or function among brain states to realize them.

The opposite seems to be true: there is nothing that is in principle unintelligible insofar as it has being, as Plato and Aristotle both thought. And we are able to be in an infinity of states of understanding, not successively but qualitatively. That is, we have the active ability to understand anything (accidents of presentation and of intelligence quotient being ignored for now). Thus, there is no arithmetical theorem we cannot understand, accidents ignored for now. Nor are there any
cess, is formally vague, no matter how far you extend it, or how minutely you describe its innermost mechanisms. The conclusion is that a physical process cannot realize an abstract function. It can at most simulate it.

What Happened to Nature? Do natural processes, say, the behavior of a freely falling body, not realize pure functions like \( d = \frac{1}{2}gt^2 \) and, where \( g = 32 \), \( d = 16t^2 \)? And is it not true that an object in empty space decreases in length in the direction it is traveling by an amount equal to \( \sqrt{1 - v^2/c^2} \)?

There are two reasons why such processes do not realize pure abstract functions of the sorts mentioned, only the second being relevant to the present discussion. First, these laws apply by idealization. What is \( \text{"the direction"} \) in which the object is traveling? There are no \( \text{"point masses."} \) That is an idealization, as is its rest mass (say, for photons or neutrinos, which are always moving at \( C \)). No object falling to earth is in a vacuum and under no gravitational attraction to other bodies. Physical phenomena often come close to our mathematizations which, of course, are invented to represent them. But those mathematizations are idealizations. That the laws are idealizations does not affect the present point.

The kind of indeterminacy I am talking about is different from that. For the incompossible functions are equally idealizations, and may differ only logically because the \( \text{"manifestation phenomena"} \) lie

---

well-formed utterances of any of the conjectured 10,000 human languages (most now lost) that we would not understand in the appropriate circumstances. But any one of those languages would require more than all the brain states. Brain states would have to be vehicles for varying content, perhaps media for thought and not the \( \text{same thing} \).

Nothing is excluded because of its subject matter. Ours is not a successive infinite capacity (if we do not exist forever) but a selective infinite capacity. That is why the brain cannot even be the organ of thought, the way the eye is the organ of sight, as Aristotle, Avicenna, Averroes, Aquinas, and many others argued; otherwise, there would be something (that might be actually) that is unintelligible. Our corporeality imposes accidental limitations on understanding, the most important of which is that our contents of judgment have to be made by dematerialization (abstraction) and our intelligence cannot directly access immaterial being (e.g., angels or God). One consequence is the indeterminacy of contingent truth (see note 17).

How the dematerialization involved in our understanding something as shape (without consideration of which thing it is, or of its particular material composition) or our understanding something as in being (without consideration of its being material) could even come about is totally beyond the resources of any known experimental or formal science.

---

beyond the actual (it being presupposed that all the actual phenomena accord with each function). So it is not a consequence of this account that there are no general and mathematizable laws of nature. Rather, just because there are general and mathematizable regularities, an object falling to the earth “in a vacuum” satisfies some incompossible function just as well as it satisfies “d = 1/2gt.” That is a consequence of the underdetermination arguments.

Now, to accept the overall argument, one does not need to deny that there are definite natural structures, like benzene rings, carbon crystals, or the structural (and behavior-explaining) molecular differences among procaine, novocaine, and cocaine. These are real structures realized in many things, but their descriptions include the sort of matter (atoms or molecules) as well as the “dynamic arrangement.” They are not pure functions.¹⁵

A musical score, say, Mahler's 2nd Symphony, can be regarded as an analog computer that determines, from any given initial sound, the successive relative sounds and their relative lengths (within conventions of intervals and length), and thus is a function from initial sound onto successor sounds; yet, from the sounds (a performance) there is not a unique score determined among incompossible ones, except by convention. So, too, when we abstract the formal structure, without matter, the physical thing (cell, molecule, gene, enzyme) or process will satisfy a logically incompossible structure just as well.

III. RETREAT FROM PEOPLE

So, to avoid the argument, someone will say:

We do not really add, either; we just simulate addition. Pure addition is just as much an idealization as $E = mc^2$. Of course, we can define such pure functions but cannot realize them; that is just a case of the many functions we can define which cannot be computed by any finite automation, or any other computer either. In a word, the fact that there is no pure addition and no pure conjunction or modus ponens is no odder than the fact that there are no perfect triangles.

We cannot really add, conjoin, or do modus ponens? Now that is expensive. In fact, the cost of saying we only simulate the pure

¹⁵ General natures (e.g., structural steel) do “have” abstract forms, but are not “pure functions.” Two humans, proteins, or cells are the same, not by realizing the same abstract form, but by a structure “solid” with each individual (but not satisfactorily described without resort to atomic components) that does not differ, as to structure or components, from other individuals. There can be mathematical abstractions of those structures, many of which we can already formulate (cf. Scientific Tables (Basel: CIBA-GEIGY, 1970)).
functions is astronomical. For in order to maintain that the processes are basically material, the philosopher has to deny outright that we do the very things we had claimed all along that we do. Yet our doing these things is essential to the reliability of our reasoning. Moreover, we certainly can, Platonistically, define the ideal functions, otherwise we cannot say definitely what we cannot do. That exposes a contradiction in the denial that we can think in pure functions, however; for to define such a function is to think in a form that is not indeterminate among incompossible forms. To become convinced that I can only simulate the recognition that two Euclidean right triangles with equal sides are congruent, I have to judge negatively with all the determinateness that has just been denied. Each Platonistic definition of one of the processes, and each description of the content of logical or arithmetical judgment, is as definite a form of thought as any of the processes being denied; and each judgment that we do not do such and such a function is as definite in form as is conjunction, addition, or any of the judgments that are challenged; otherwise, what is denied would be indeterminate. It is implausible enough to say we do not really add or conjoin. It is beyond credibility to say we cannot definitely deny that we add, conjoin, assert the congruence of triangles, or define particular functions, like conjunction.

The final and greatest cost of insisting that our judgments are not more determinate as to pure functions than physical processes can be, is that we can do nothing logical at all, and no pure mathematics either. Now, who believes that?

There is not some parallel evidential indeterminacy between our activities and those of a machine whereby we cannot be sure what either is doing. The machine cannot in principle add. We can be sure of that. And we can, and do add, and conjoin and reason syllogistically. We can be sure of that, too.

Someone rejoins, “So you say. But we might be just simulating.” The rejoinder defeats itself. By its presumption, it grants the force of the argument as a whole, that there are pure functions and that, if certain thought processes were physical processes or functions among them, they would not be formally determinate. It merely asserts as a counterpossibility that I may think I am adding, etc., when I am only simulating a pure function. But to think I am adding

I think Kripke (op. cit., pp. 21, 65, and 71) interprets what he regards as indeterminacy as to whether I meant plus or quus as the basis for alleging an indeterminacy about what I do. (“There is no fact of the matter.”) I say this gets the explanatory order backward and invites mistaken conclusions.
or conjoining, with a clear idea of what that is, is to perform a pure function in that very thought, whether it is true or not.

Besides, such counterpossibilities require an ontological status for the pure functions simulated. We think of them and even define them. If that is so, then the thoughts and definitions cannot be indeterminate among incompatible functions because no definite function would then be defined by such thinking. So those function-determining thoughts cannot themselves be just simulations but have to realize pure functions, e.g., “defining addition,” “conceiving modus ponens.”" Hence, in order to be mistaken in a certain way, I have to think in exactly the way that cannot be entirely physically realized.

To say we may not know whether we are adding, when we are, or squaring, when we are, is actually to grant that we might perform the determinate thought function that cannot be wholly physical, and thus to grant the whole argument. Similarly, to say, “We do not know whether we ever perform a formally determinate function,” is to say either (a) we are in a cognitive state, “uncertainty as to whether we are really adding, squaring, or conjoining,” although we do not experience uncertainty, when we produce sums, squares, and simple arguments; or (b) we are always mistaken when we are certain we are adding, conjoining, etc., because at most we simulate.

Now, the first option also concedes the main argument because it postulates uncertainty when we actually do add, etc. The second postulates mistakes about what we are doing, and thus concedes the main argument, too: that there are such definite functions for which the only locus must be in thought. Any other answer will leave the pure function without any logical space (locus). When we are certain we are adding, we are always wrong. But that reasoning will hold for whatever we do. Thus, we are always wrong about what we think we are doing, when we think we are doing something definite enough to be a pure function. To suppose we can think definitely enough about functions to be wrong about what we are doing concedes the supposition of the argument again. Now the doubt has spread to include every pure function: asserting, questioning, objecting, stating, reporting, as well as adding, squaring, and conjoining. The doubt has even spread to include the very repeating of what I take, mistakenly, to be my argument and to make it indefinite whether you are actually denying or disputing my conclusion. Moreover, the cost extends to particular pure functions, specified by content: “adding three and three,” “judging that Greeks are courageous,” “doubting whether philosophy is scientific,” “reading a paper,”
"thinking this writer is mad." Such an epidemic of doubt, without any effect on one's own certainty, must involve a mistake.

If we are always only simulating when we think we are doing something formally definite, then it is never determinate what we are doing at all. That requires that we are never doing such definite things at all. That is expensive, because there is no place for logic or mathematics or any other formal thinking at all; we cannot even "castle" in chess, but only "simulate" it, without any explanation for what "it" is or what its status is ontologically. Saint Augustine similarly objected to a "verisimilitude" account of truth in Contra Academicos. The relation of simulation will not be definable without the prior notion of pure functions.

If we can agree that either (1) we do have such definite thought processes as I described, cases of conjunction, determinate among all incompatible functions, and that they cannot wholly be physical processes (or functions among physical processes only), or (2) we never perform such processes but at most simulate them, then I am content. For I shall then wait for the counterattack to support (2), the one that explains the status of all those functions I cannot really perform and only think I can define (for to define one is to perform another one), and, in particular, explains the success of mathematics and pure logic, especially natural deduction systems and the proofs of completeness of propositional calculus, and offers a worked-out contrast between adding (which no one, apparently, can do) and simulating adding.

IV. FUNCTIONAL STATES

Kripke seemed to realize that functionalism would fail because "any concrete physical object can be viewed as an imperfect realization of many machine programs" (op. cit., pp. 36–7, n24). But it looks to me as if he was about to draw the wrong conclusion, when he said "taking a human organism as a concrete object, what is to tell us WHICH program he should be regarded as instantiating? In particular, does he compute 'plus' or 'quus'?" He should have concluded that, if a human is only a "concrete physical object," then nothing determines, at a certain level of refinement, which program it instantiates because it instantiates none; whereas humans do add, define, and so forth, and are thus not just concrete physical objects.

If a "thought process," say, adding, were a function linking actual physical states to "subsequent" physical states, then whatever the pattern of inputs to outputs, there are incompossible functions that link the states equally well. In that case, we could not really add. Nor could we deny that we add precisely. Since we can add, we know our
thought process is not the same as any function among brain states because no such function is determined (the way two points determine a line) by physical states.

The very step toward generality to escape the inconveniences of identifying an abstract process with a particular physical process, say, mechanical addition (with the inconvenience that there could be no electronic addition), creates the situation where incompossible general functions equally well “explain” the succession of physico-cognitive states, and thus discloses that no one function is realized to the exclusion of all the others at the physical level, and thus no pure function is realized at all. That guarantees that functions among physical states (in a process) are not the thought states because there are no determinate functions realized among physical states, when the form of thought is determinate. No real process of adding is identical with any process that equally well realizes an incompossible function. Consequently, “adding” is not a physical process or function among physical states either. Besides, the functors in such functions are not physical either. For, of course, it is numbers we add, not numerals.

V. ALL THOUGHT IS ABSTRACT

The main argument is that some thought is determinate, among incompossible functions, the way no physical process, series of processes, or physically determined function among processes can be. The result is that such thought is never identical with any physical process or function. (Nor can it really be such a physical process or function either, though it may, for all we have said, have a material medium, like speech.)

The full generalization that all thought is determinate that way is harder to make cogent, because it rests on one’s recognizing that, whatever thinking we do, whether simple assertion or hoping or wanting or intending (over the whole family of things each of those can be, according to its particular content on a particular occasion) is such that, in order to do that, we have to do what is the same for an infinity of other cases (sorted by content) that do not happen. For someone else might have thought or said or believed or felt the same in a way definite among incompossibles. So, any thinking at all is of general “form,” just as is adding, conjoining, reasoning validly, and squaring.

By its nature, thinking has “other cases” and is therefore always of a definite form (which may not be articulable by us, as are mathematical and logical forms). Asserting (in any one of its senses) cannot be “halfway” between opposed forms; it would not be asserting
then. And so on, for every form of thinking. But no physical process or sequence of processes or function among processes can be definite enough to realize ("pick out") just one, uniquely, among incompossible forms. Thus, no such process can be such thinking.

The conclusion is that no physical process or sequence of processes or function among physical processes can be adding, squaring, asserting, or any other thinking at all.\(^{17}\)

JAMES ROSS

University of Pennsylvania

\(^{17}\) All thought, as content, is immaterial in two other ways. (1) It lacks the transcendent determinacy of the physical. A true judgment, "someone is knocking on my door," requires for its physical compliant reality a situation with an infinity of features not contained (or logically implied) in the true judgment. Thus, an infinity of determinate but incompossible physical situations could make the same statement true. (2) Any physical-object truth requires its truth-making reality to overflow the thought infinitely in the detail of what obtains. So every compliant reality is infinitely more definite than anything contingently true we can say about it. It takes a lakeful of reality for one drop of truth.

A second argument: Products of physical processes are transcendentally determinate. But no product of the understanding has an infinity of content, not contained therein logically. So no physical product can ever be such a content of the understanding.

Some thinking is as much physical as it is immaterial. My walking, as an action, is as much a mode of thought as it is a mode of movement; yet no movement, however complex, could ever make a thought.

Leibniz says in section 17 of the Monadology (in Philosophical Papers and Letters, Leroy Loemker, ed. and trans. 2nd ed. (Dordrecht: Reidel, 1969), p. 644) that, if perception were supposed to be produced by a machine, we could make the machine on large scale and walk around in it like a mill; we would never find a perception, only the movements of wheels, gears, and pulleys. Similar reasoning is given in Leibniz's Conversation of Philarete and Ariste (Loemker, p. 623). I thank Margaret Wilson for pointing these passages out to me.

A third argument: The present cases concern the definiteness of the form of the thinking. A third, parallel argument can be constructed from the definiteness of the content of thought, that thought is definite among incompossible contents in a way no physical process can ever be. Similar underdetermination arguments apply.

Machines do not process numbers (though we do); they process representations (signals). Since addition is a process applicable only to numbers, machines do not add. And so on for statements, musical themes, novels, plays, and arguments.